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## Surface Coverage of Filter Medium in Deep Bed Filtration: Mathematical Modeling and Experiments

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### ABSTRACT

This paper highlights the importance of surface coverage in modeling the removal of particles in deep bed filtration. A model that considers the saturation of sites on which particle deposition occurs is used. Experimental results obtained with monodispersed suspensions of 0.46 and 0.816  $\mu\text{m}$  latex particles at different influent concentrations and ionic strengths were used to calculate the fraction of filter grain surface ( $\beta_1$ ) on which actual particle deposition occurs. This will be useful in evaluating the filter performance in terms of the utilization of available surface area of the filter medium. Further, the level of dendrite formation of particles on filter grains during filtration is expressed in terms of  $\beta_1$  and the specific surface coverage,  $\theta_T$  (the fraction of a filter grain surface that is covered by particles at time  $T$ , assuming that the filter grain is covered by a monolayer of particles). This can be used to compare the contribution of deposited particles in the removal efficiency of deep bed filtration for suspensions with different physical and chemical characteristics.

**Key Words.** Deep bed filtration; Dendrites; Filter grain surface; Ionic strength; Latex particles; Mass concentration; Number concentration; Monolayer deposition; Specific surface coverage; Surface area concentration

### INTRODUCTION

Particle removal in deep bed filtration is physicochemical in nature and depends on the physical and chemical characteristics of particles, filter grain, water, and chemicals used. A number of mathematical models have been de-

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veloped to calculate particle capture in the filter (1–14). One approach has been to establish the mathematical model based on the analogy between adsorption and filtration (4, 9). In this approach the effective surface coverage of filter grains by the particles is assessed using the mathematical model for various filtration conditions. The effect of influent concentration and ionic strength of the solution on the extent of surface coverage was studied.

## MATHEMATICAL FORMULATIONS

In this study the model developed based on the analogy between adsorption and filtration was used. A brief explanation of the model and the calculation method of specific surface coverage are given below. Details can be found elsewhere (9).

### Mathematical Model

This model considered that there is a saturation of sites on which particle deposition occurs. Deposition sites on both the filter grain and on the retained particles are taken into account. When the deposition exceeds a limit, there is no more retention of particles within the layer under consideration. Two limits were imposed: one on the maximum number of particles that can be attached directly onto the filter grain and the other on the maximum number of particles that can be retained both on the filter grain and on the particle collectors associated with that filter grain.

The maximum number of particles deposited directly onto the surface of the filter grain is given by

$$\frac{\text{surface area of filter grain}}{\text{cross-sectional area of particle to be removed}}$$

i.e.,

$$4[d_c/d_p]^2$$

where  $d_c$  and  $d_p$  are the diameters of filter grain and particle, respectively. The above expression is based on the fact that the whole surface of a filter grain is active in the deposition of particles. But in actual fact, only a portion ( $\beta_1$ ) of the surface of the grain will be available for the direct deposition of the particles because of the shadow effect. The maximum number of particles ( $N_{\max}$ ) retained on the surface of a filter grain can therefore be given by

$$N_{\max} = 4\beta_1[d_c/d_p]^2 \quad (1)$$

The maximum number of particles associated with a filter grain (i.e., the number of particles deposited on the filter grain and on the particle collectors associated with that filter grain) is defined by defining a “limiting poros-



ity" ( $\varepsilon^*$ ) term. The limiting porosity can be related to ultimate specific deposit ( $\sigma_{\text{ult}}$ ) by

$$\varepsilon^* = \varepsilon_0 - \sigma_{\text{ult}}/(1 - \varepsilon_d) \quad (2)$$

where  $\varepsilon_0$  is the porosity of a clean filter bed and  $\varepsilon_d$  is the porosity of deposit. The porosity of the filter  $\varepsilon$  is reduced with the progressive deposition of particles (Eq. 3), and there will not be any further deposition in the particular layer considered when its limiting value is reached.

The porosity at a given time ( $\varepsilon$ ) can be related to the specific deposit ( $\sigma$ ) at that particular time as follows:

$$\varepsilon = \varepsilon_0 - \sigma/(1 - \varepsilon_d) \quad (3)$$

Particle deposition in deep bed filtration is governed both by particle transport toward filter grains and attachment thereafter. First, particles in suspension are transported near filter grains by mechanisms such as sedimentation, interception, diffusion, inertia, and hydrodynamic effect. This is followed by the attachment of particles onto filter grains or particles already deposited onto filter grains. The attachment depends on the surface forces acting between particles and filter grains when their separation distance becomes of the order of a nanometer. The surface forces will be altered continuously with the progressive deposition of particles onto filter grains. Thus, the rate of striking of particles onto filter grains (or attachment) depends on the available surface area or deposition sites. If one assumes that the rate of particles striking and attaching directly to the surface of the filter grain is proportional to the available surface area of the filter grain, then the amount of particles attached on the filter grain at a particular time  $t$  is given by (see Appendix for the derivation of Eq. 4)

$$\alpha\eta[(N_{\text{max}} - N_1)/N_{\text{max}}] (\pi/4) d_c^2 Un \quad (4)$$

where  $\alpha$  is the attachment coefficient between particles and the filter grain,  $\eta$  is the contact efficiency of a filter grain,  $N_1$  is the number of particles directly attached on the filter grain at a given time  $t$ ,  $U$  is the filtration velocity, and  $n$  is the particle concentration at a given time and depth. Here, it is assumed that at any given time, a fixed fraction ( $\gamma$ ) of the total number of particles deposited ( $N_2$ ) is attached directly to the filter grain, i.e.,

$$N_1 = \gamma N_2 \quad (5)$$

Further, if one assumes that the particle deposition on the particle collector is proportional to the available storage space (the available space is proportional to  $\varepsilon - \varepsilon^*$ ), then the efficiency of the removal of particles by a particle collector can be given as

$$\alpha_p\eta_p[(\varepsilon - \varepsilon^*)/(\varepsilon_0 - \varepsilon^*)](\pi/4)d_p^2 Un \quad (6)$$



where  $\alpha_p$  is the attachment coefficient between particles and particles collectors, and  $\eta_p$  is the contact efficiency of a particle collector. Combining the above three equations, the removal efficiency of a single collector (filter grain and associated particle collectors on it),  $\eta_r$ , can be given as

$$\begin{aligned}\eta_r = \alpha\eta[(N_{\max} - \gamma N_2)/N_{\max}] \\ + \alpha_p\eta_p N[d_p/d_c]^2[(\varepsilon - \varepsilon^*)/(\varepsilon_0 - \varepsilon^*)]\end{aligned}\quad (7)$$

where  $N$  is the number of particle collectors in a unit volume. The rate of change of particle collectors at any time can be calculated as follows by assuming that a fraction,  $\beta$ , of retained particles acts as particle collectors:

$$\partial N/\partial t = \beta(\pi/4) d_c^2 U n \eta_r \quad (8)$$

From the mass balance of the suspension in a small volume element of the filter bed, the following equation can be written:

$$\partial n/\partial t + U \partial n/\partial L + (\frac{3}{2})[(1 - \varepsilon_0)/d_c] U n \eta_r = 0 \quad (9)$$

where  $L$  is the filter depth. Considering  $\eta_r$  and  $n$  as step functions of time and assuming steady state ( $\partial n/\partial t = 0$ ), Eq. (9) can be written

$$\partial n/\partial L = -(\frac{3}{2})[(1 - \varepsilon_0)/d_c] n \eta_r \quad (10a)$$

By integrating Eq. (10a) between the limits 0 and  $\Delta L$  for  $L$ , and  $n_0$  and  $n_i$  for  $n$ , the following expression can be obtained:

$$n_i/n_0 = \exp[-(\frac{3}{2})(1 - \varepsilon_0) \eta_{r(i-1)} (\Delta L/d_c)] \quad (10b)$$

where  $n_i$  is the particle concentration at the  $i$ th time step,  $n_0$  is the influent particle concentration, and  $\Delta L$  is the increment in filter depth. The rate of change of the number of particle collectors at the  $(i - 1)$ th time step is given by

$$(N_i - N_{i-1})/\Delta t = \beta(\pi/4) d_c^2 U n_{i-1} \eta_{r(i-1)} \quad (11)$$

Therefore, the total number of particle collectors up to the  $i$ th time can be given by

$$N_1 = \beta(\pi/4) d_c^2 U \sum_{i=1}^t n_{i-1} \eta_{r(i-1)} \Delta t \quad (12)$$

Substituting Eqs. (10) and (12) in Eq. (7), the removal efficiency at the  $i - 1$ th time interval can be obtained as follows:

$$\begin{aligned}\eta_{r(i-1)} = \alpha\eta[(1 - \gamma N_2/N_{\max})] + \alpha_p\eta_p\beta \\ \times (\pi/4) [d_p]^2 U [(\varepsilon - \varepsilon^*)/(\varepsilon_0 - \varepsilon^*)] \sum_{i=1}^t n_0 \eta_{r(i-1)} \\ \times \Delta t \exp[-(\frac{3}{2})(1 - \varepsilon_0) \eta_{r(i-1)} (\Delta L/d_c)]\end{aligned}\quad (13)$$



The value of the ultimate specific deposit should be known in order to calculate the maximum pore blockage or the porosity of the clogged bed ( $\varepsilon^*$ ). The procedure to evaluate  $\varepsilon^*$  is given in the following section.

### Calculation Procedure to Evaluate the Porosity $\varepsilon^*$ of a Clogged Bed

The porosity deposit ( $\varepsilon_d$ ), ultimate specific deposit ( $\sigma_{ult}$ ), and ultimate porosity of the filter bed ( $\varepsilon^*$ ) were experimentally measured by Vigneswaran and Chang (10). Since the ratio of effluent to influent concentration ( $C/C_0$ ) will take a very long time to reach the value of unity, the filter run is terminated when the postbreakthrough stage starts. The postbreakthrough stage is defined as the period where the effluent concentration remains almost constant after a period of deterioration of effluent quality (i.e., after the breakthrough period). The deposit is washed out (when the postripening period is reached) and allowed to settle down up to a permanent volume ( $V_d$ ). The dry weight of the deposit ( $W_d$ ) is then measured. From these values,  $\varepsilon_d$ ,  $\sigma_{ult}$ , and  $\varepsilon^*$  can be calculated using the following equations:

$$\varepsilon_d = 1 - (W_d/\rho_d)/V_d \quad (14)$$

$$\sigma_{ult} = (W_d/\rho_d)/V_f \quad (15)$$

where  $\rho_d$  is the density of the deposit and  $V_f$  is the volume of the filter bed. Here, the ultimate specific deposit,  $\sigma_u$ , is defined as the deposit up to the period of postbreakthrough per unit volume of filter. Then  $\varepsilon^*$  can be given as follows (Eq. 2):

$$\varepsilon^* = \varepsilon_0 - (\sigma_{ult})/(1 - \varepsilon_d)$$

$\varepsilon_d$  was taken as 0.8 in this study, which is consistent with the values used in previous work with the same suspensions (9).

### Specific Surface Coverage

Specific surface coverage ( $\theta$ ) in a time interval ( $\Delta t$ ) is defined as the ratio between the surface area of filter grains that is covered by deposited particles in a unit bed volume in a time interval of  $\Delta t$  and the total surface area of filter grains in a unit bed volume. Thus, for a monolayer coverage of particles onto filter grains,  $\theta$  at time  $T$  from the beginning of filtration ( $\theta_T$ ) can be given as (13, 15)

$$\theta_T = \{(\pi a_p^2)UN_0 a_c/[3L(1 - \varepsilon)]\} \int_0^T (1 - N_{out}/N_0)dt \quad (16)$$

where  $a_c$  and  $a_p$  are the radii of filter grains and particles, respectively, and  $N_0$  and  $N_{out}$  are the number concentrations of particles in the influent and in the effluent, respectively. In the model discussed in the previous section,  $\theta_T$  is de-



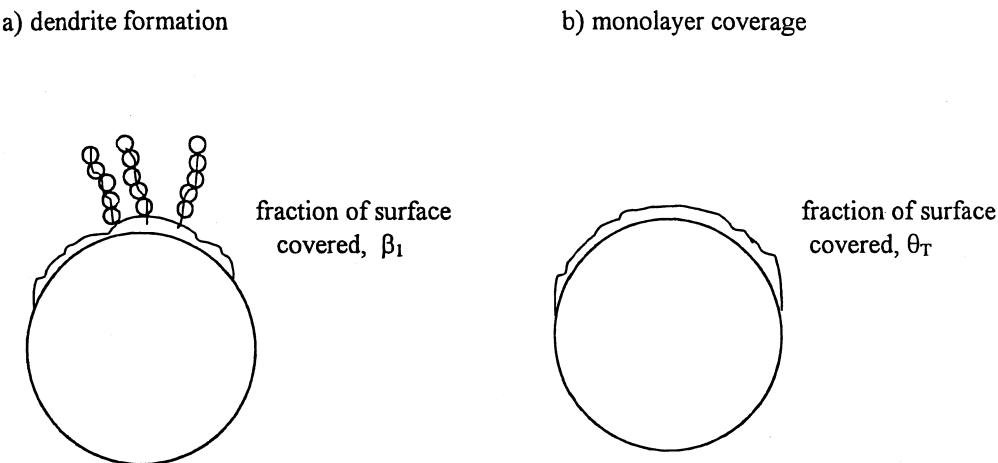


FIG. 1 Comparison of dendrite formation and monolayer coverage.

notes the fraction of filter grain surface that is directly covered by the particles. These particles and the particles deposited in dendrite forms amount to the total deposition of particles on a filter grain (Fig. 1a). On the other hand, when monolayer coverage of particles onto filter grains is assumed, the specific surface coverage  $\theta_T$  denotes the fraction of filter grain surface that is directly covered by the particles, and these particles amount to the total deposition (Fig. 1b). Thus, if monolayer coverage had occurred, the value of  $\theta_T$  should exceed the value of  $\beta_1$  after a certain time of filtration. In the case of dendrite formation, the difference between  $\theta_T$  and  $\beta_1$  should represent the amount of dendrites formed.

However, monolayer coverage of filter grains is improbable for most filtration conditions, as the particles will arrive in a random manner and the coverage will build up on a declining basis with some multiple coverage occurring at the same time, possibly with the chain's formation (dendrite). In this case,  $\theta_T$  will be larger than  $\beta_1$  when  $T$  exceeds a particular time. From that time onward, the difference between  $\theta_T$  and  $\beta_1$  can be used as a measure of the amount of dendrite formed.

## EXPERIMENTAL

Latex particles of 0.460 and 0.816  $\mu\text{m}$  were used to prepare monodispersed suspensions of predetermined concentrations. Spherical glass beads of 0.175 mm were used as the filter medium and were packed into the cylindrical filter column to a specified depth. A downward filtration velocity of 2.5 m/h was used. Effluent turbidity ( $C$ ) was measured at predetermined time intervals using a HACH turbidimeter. The zeta potential value of the influent was mea-



TABLE 1  
Experimental Conditions

Effect of	Particle size ( $\mu\text{m}$ )	Influent concentration (mg/L)	Ionic strength, $\log[\text{KCl}]$ (M)	Filter depth (cm)
Concentration	0.460	1.0, 2.0, 3.0, 4.0, 5.0, 7.5, 10.0	-2.0	10
	0.816	1.77, 5.32, 5.58, 8.87, 16.74, 27.91	-2.0	10
Ionic strength	0.460	5.0	-6.0, -4.0, -3.0, -2.5, -2.0	10
	0.816	5.0	-6.0, -4.0, -3.0, -2.5, -2.0	10
Filter depth	0.460	1.0, 2.0, 3.0, 4.0, 5.0, 10.0	-2.0	5
	0.460	1.0, 2.0, 3.0, 4.0, 5.0, 7.5, 10.0	-2.0	10

sured using a DELSA 440. There was no particle aggregation observed in the range of influent concentration and the ionic strength used. The experimental conditions are summarized in Table 1.

## RESULTS AND DISCUSSION

Experimental results obtained and the corresponding model predictions are shown in Figs. 2(a) through 2(e). Corresponding values of the model coefficient  $\beta_1$  are discussed in subsequent sections. Other model parameters were kept constant at the following values:  $\alpha_p\beta = 0.0001$ ,  $\varepsilon^* = 0.30$ , and  $\gamma = 0.22$ .

### Influent Concentration

#### *Effect of Influent Concentration on $\beta_1$*

From Figs. 2(a), (b), and (c) it can be seen that the removal efficiency of 0.460 and 0.816  $\mu\text{m}$  particles increases during the transient stages of filtration when the influent mass concentration of monodispersed suspensions of 0.460 and 0.816  $\mu\text{m}$  particles was increased up to around 5 mg/L. When the concentration of both monodispersed suspensions of 0.460 and 0.816  $\mu\text{m}$  particles was increased in this range, the effluent concentration profiles were fitted with increasing values of  $\beta_1$ . The higher the concentration (up to 5 mg/L), the higher the coverage fraction of a filter grain surface (Table 2). When the concentrations of those monodispersed suspensions were increased above 5 mg/L, the working stage removal efficiency was found to decrease. Corre-



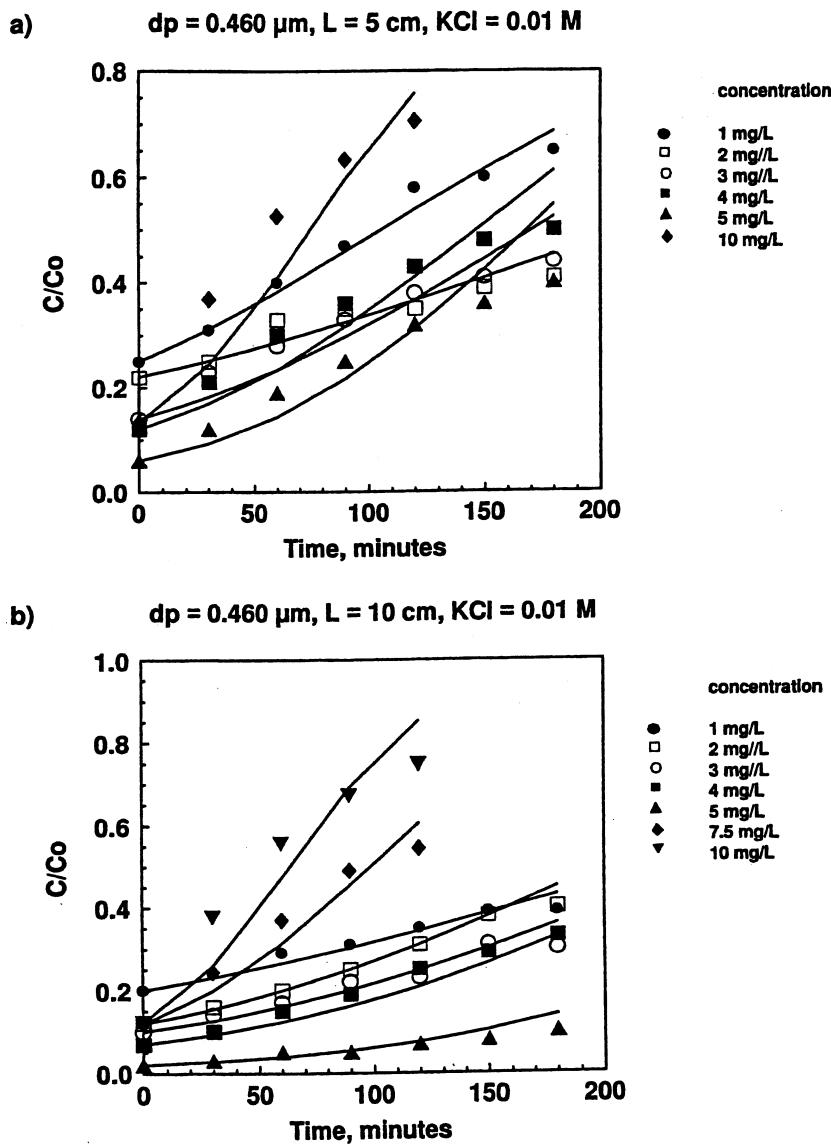


FIG. 2 Experimental data with model prediction. Symbols denote experimental data and lines denote model prediction.

spondingly, for both monodispersed suspensions the values of  $\beta_1$  decreased when the influent concentration was increased above 5 mg/L.

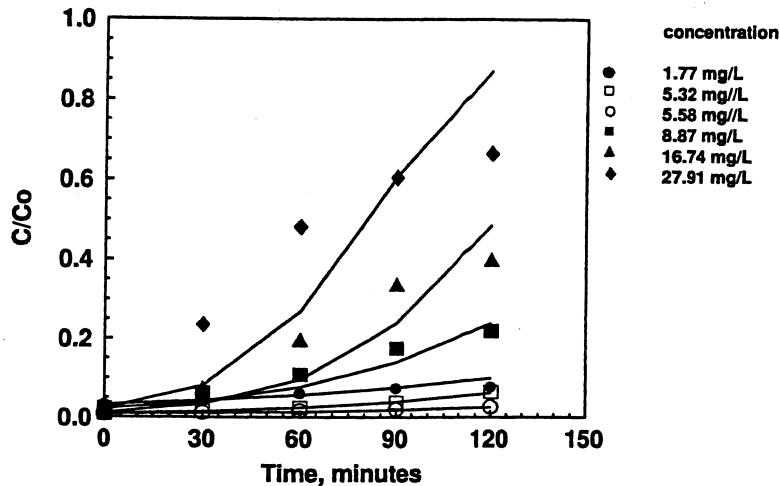
Influent concentration can also be measured as surface area concentration ( $A_0$ ) and number concentration ( $N_0$ ). The relationship between  $A_0$  and  $N_0$  with mass concentration  $C_0$  can be written as

$$N_0 = C_0 / \left[ \left( \frac{4}{3} \right) \pi a_p^3 \rho \right] \quad (17)$$

$$A_0 = (4\pi a_p^2) N_0 \quad (18)$$



c)  $d_p = 0.816 \mu\text{m}$ ,  $L = 10 \text{ cm}$ ,  $\text{KCl} = 0.01 \text{ M}$



d)  $d_p = 0.460 \mu\text{m}$ ,  $L = 10 \text{ cm}$ ,  $C_0 = 5 \text{ mg/L}$

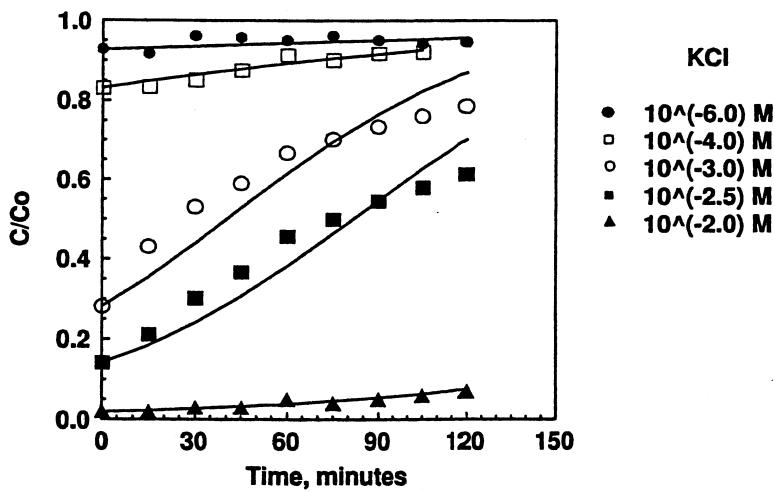


FIG. 2 Continued.

(continued)

where  $a_p$  is the radius of particles and  $\rho$  is the density of particles. Table 3 shows the relationship among mass, surface area, and number concentrations of 0.460 and 0.816  $\mu\text{m}$  particles. These values were calculated using the above two equations.

For monodispersed suspensions of 0.460 and 0.816  $\mu\text{m}$  particles having the same surface area concentrations of less than  $0.37 \text{ cm}^2/\text{mL}$ , the fraction of a filter grain surface ( $\beta_1$ ) on which actual particle deposition occurs is larger for 0.816  $\mu\text{m}$  particles (Table 4). Thus, for equal area concentrations of less than  $0.37 \text{ cm}^2/\text{mL}$ , the removal of 0.816  $\mu\text{m}$  particles is better than that of 0.460  $\mu\text{m}$  particles.



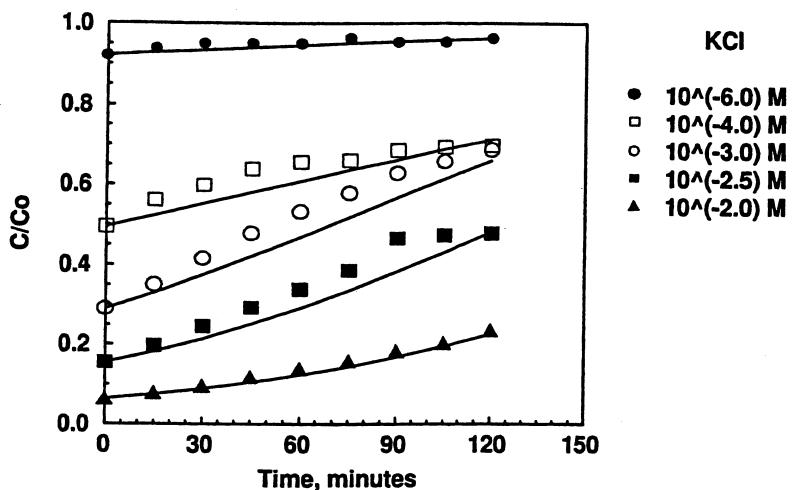
e)  $dp = 0.816 \mu\text{m}$ ,  $L = 10 \text{ cm}$ ,  $Co = 5 \text{ mg/L}$ 

FIG. 2 Continued.

$\mu\text{m}$  particles (i.e., smaller  $C/C_0$  values for  $0.816 \mu\text{m}$  particles compared to  $0.460 \mu\text{m}$  particles during filtration).

For equal surface area concentrations between  $0.37$  and  $0.62 \text{ cm}^2/\text{mL}$ ,  $\beta_1$  is larger for  $0.460 \mu\text{m}$  particles and correspondingly the removal of  $0.460 \mu\text{m}$  particles is better than that of  $0.816 \mu\text{m}$  particles.

Similarly, for monodispersed suspensions of  $0.460$  and  $0.816 \mu\text{m}$  particles having the same number concentrations of less than  $56.06 \times 10^6/\text{mL}$ , the frac-

TABLE 2  
Value of  $\beta_1$  at Different Mass Concentrations of Monodispersed Suspensions of  $0.460$  and  $0.816 \mu\text{m}$  Particles

0.460 $\mu\text{m}$		0.816 $\mu\text{m}$	
Concentration (mg/L)	$\beta_1$	Concentration (mg/L)	$\beta_1$
1.0	0.003	1.77	0.004
2.0	0.005	5.32	0.010
3.0	0.009	5.58	0.018
4.0	0.012	8.87	0.010
5.0	0.020	16.74	0.012
7.5	0.009	27.91	0.010
10.0	0.008		



TABLE 3  
Variation of Surface Area and Number Concentration with the Influent Concentration

Influent concentration (mg/L)	0.460 $\mu\text{m}$ Particles		0.816 $\mu\text{m}$ Particles	
	cm $^2$ /mL	No. $\times 10^6$ /mL	cm $^2$ /mL	No. $\times 10^6$ /mL
1.00	<b>0.124</b>	<b>18.69</b>	0.070	3.35
1.77	0.220	33.08	<b>0.124</b>	5.93
3.00	<b>0.373</b>	<b>56.06</b>	0.210	10.04
5.00	<b>0.621</b>	<b>93.44</b>	0.350	16.74
5.32	0.661	99.42	<b>0.373</b>	17.81
5.58	0.693	104.27	0.391	<b>18.68</b>
8.87	1.102	165.75	<b>0.621</b>	29.69
16.74	2.080	312.82	1.172	<b>56.04</b>
27.90	3.466	521.37	1.954	<b>93.40</b>

\* Equal surface area concentrations and equal number concentrations of 0.46 and 0.816  $\mu\text{m}$  particles are shown in bold.

tion of a filter grain surface ( $\beta_1$ ) on which actual particle deposition occurs is larger for 0.816  $\mu\text{m}$  particles. For equal number concentrations between  $56.06 \times 10^6/\text{mL}$  and  $93.43 \times 10^6/\text{mL}$ ,  $\beta_1$  is larger for 0.460  $\mu\text{m}$  particles. Thus, for equal number concentrations less than  $56.06 \times 10^6/\text{mL}$ , the removal of 0.816  $\mu\text{m}$  particles is better than that of 0.460  $\mu\text{m}$  particles, and for equal number concentrations between  $56.06 \times 10^6/\text{mL}$  and  $93.43 \times 10^6/\text{mL}$ , removal of 0.460  $\mu\text{m}$  particles is better than that of 0.816  $\mu\text{m}$  particles.

### Effect of Influent Concentration on $\theta_T$

Figures 3 and 4 show the relationship between the specific surface coverage  $\theta_T$  and  $C/C_0$  for equal surface area concentrations and equal number concentrations, respectively. These figures imply that for both monodispersed suspensions, if the filter grains are covered by a monolayer of particles, then

TABLE 4  
Values of  $\beta_1$  at Different Surface Area and Number Concentrations of the Monodispersed Suspensions of 0.460 and 0.816  $\mu\text{m}$  Particles

Surface area concentration (cm $^2$ /mL)	$\beta_1$		Number concentration (no. $\times 10^6$ /mL)	$\beta_1$	
	0.460 $\mu\text{m}$	0.816 $\mu\text{m}$		0.460 $\mu\text{m}$	0.816 $\mu\text{m}$
0.124	0.003	0.004	18.69	0.003	0.018
0.373	0.009	0.010	56.06	0.009	0.012
0.621	0.020	0.010	93.44	0.020	0.010



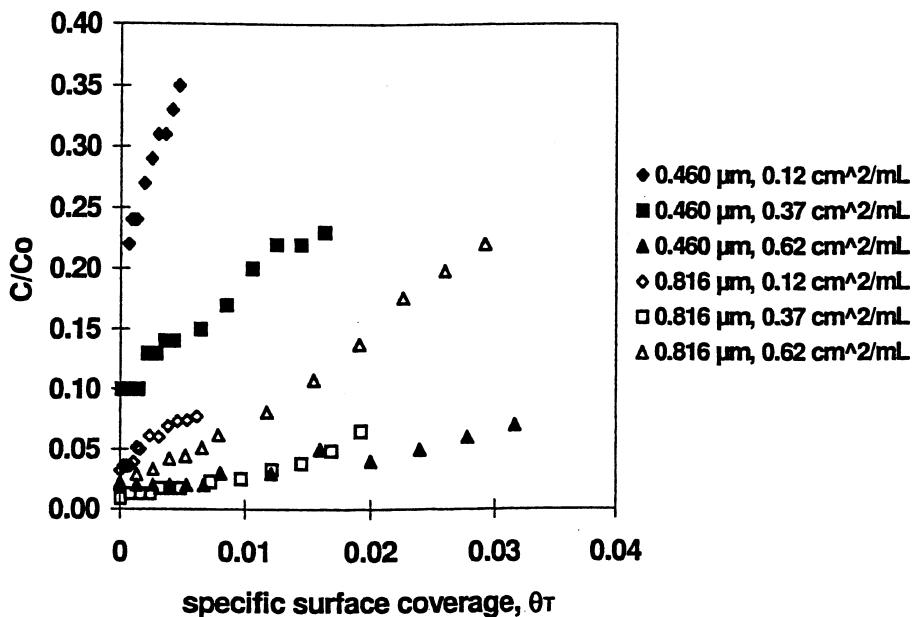


FIG. 3 Relationship between specific surface coverage and  $C/C_0$  for monodispersed suspensions of 0.460 and 0.816  $\mu\text{m}$  particles at equal surface area concentrations ( $L = 10 \text{ cm}$ ,  $\text{KCl} = 0.01 \text{ M}$ ).

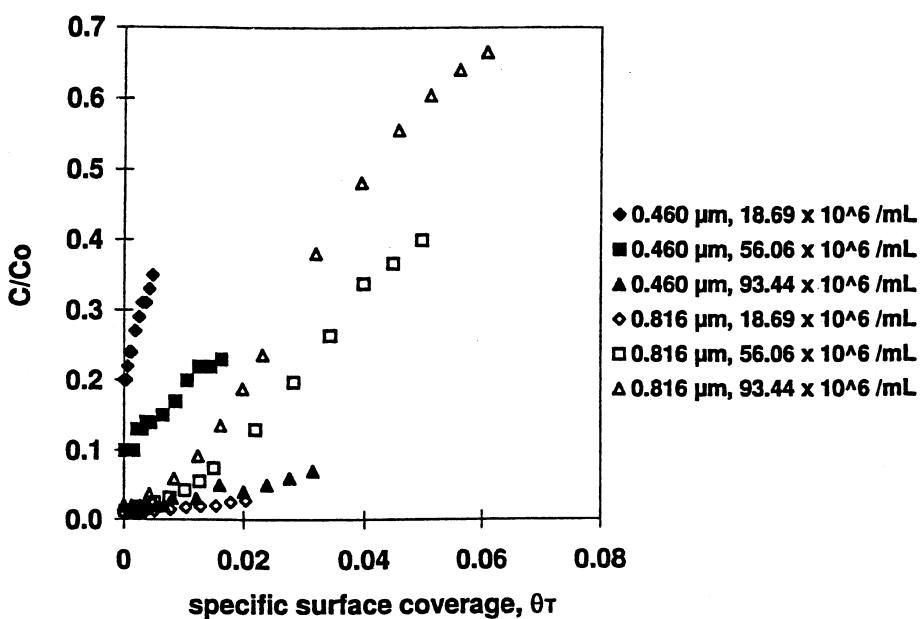


FIG. 4 Relationship between specific surface coverage and  $C/C_0$  for monodispersed suspensions of 0.460 and 0.816  $\mu\text{m}$  particles at equal number concentrations ( $L = 10 \text{ cm}$ ,  $\text{KCl} = 10^{-2} \text{ M}$ ).



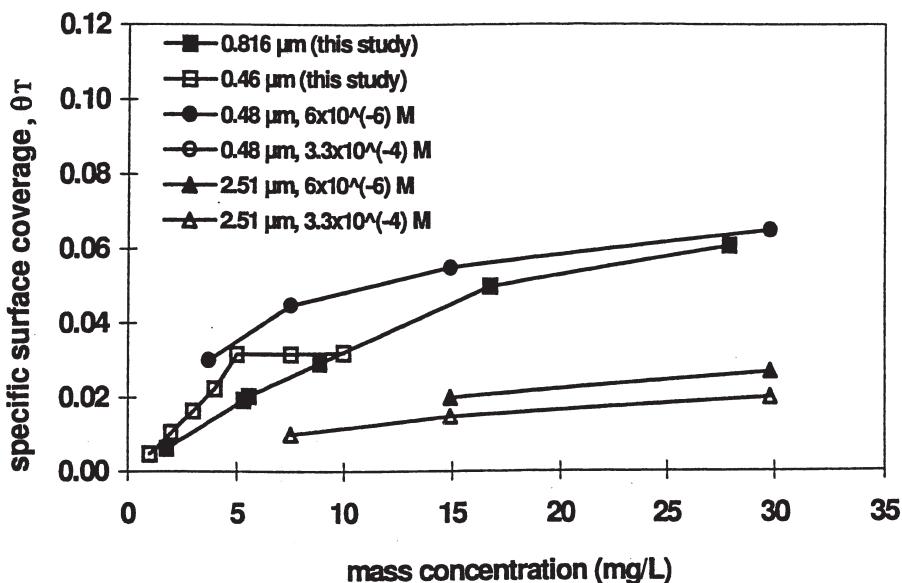


FIG. 5 Variation of specific surface coverage with influent concentration [this study:  $T = 120$  minutes,  $L = 10$  cm,  $KCl = 0.01$  M,  $U = 2.5$  m/h; previous study by Liu et al. (14):  $T = 80$  minutes,  $L = 10$  cm for  $0.48 \mu\text{m}$  particles and  $14.2$  cm for  $2.51 \mu\text{m}$  particles,  $U = 3.6$  m/h].

at a given time  $T$  the value of  $\theta_T$  is larger for a suspension having a larger surface area concentration or number concentrations. However, when the number concentrations of  $0.460$  and  $0.816 \mu\text{m}$  particles are kept constant in the monodispersed suspensions,  $\theta_T$  is larger for  $0.816 \mu\text{m}$  particles compared to  $0.460 \mu\text{m}$  particles. But for monodispersed suspensions with equal surface area concentrations,  $\theta_T$  was similar for both  $0.816$  and  $0.460 \mu\text{m}$  particles.

The values obtained for  $\theta_T$  from the filtration experimental results (at 120 minutes of filtration) for  $0.460$  and  $0.816 \mu\text{m}$  particles at  $0.01$  M KCl ionic strength are shown in Fig. 5. The results from a previous study with two different sizes of particles at two different ionic strength values are also presented for comparison (14) (at  $T = 80$  minutes). Suspensions having larger particles tend to show less specific surface coverage at a particular time when compared with suspensions having smaller particles of the same mass concentration.

### Comparison between $\beta_1$ and $\theta_T$

The values of  $\theta_T$  (at  $T = 120$  minutes of filtration) obtained for different concentrations are presented in Tables 5 and 6. For monodispersed suspensions of  $0.460$  and  $0.816 \mu\text{m}$  particles having an equal surface area concentration, the difference between  $\theta_T$  and  $\beta_1$  ( $\theta_T - \beta_1$ ) is larger for  $0.816 \mu\text{m}$  particles in the concentration range between  $0.37$  and  $0.62 \text{ cm}^2/\text{mL}$  (Fig. 6). For this concen-



TABLE 5  
Effect of Influent Concentration of Monodispersed Suspension  
of 0.460  $\mu\text{m}$  Particles on  $\theta_T$  (at  $T = 120$  minutes)

Concentration (mg/L)	$\theta_T$ at $T = 120$ minutes	
	$L = 5$ cm	$L = 10$ cm
1.0	0.008	0.005
2.0	0.018	0.010
3.0	0.028	0.016
4.0	0.038	0.022
5.0	0.053	0.032
7.5	—	0.032
10.0	0.067	0.032

tration range it is reasonable to conclude that 0.816  $\mu\text{m}$  particles did not deposit directly onto a filter grain surface to the extent with which the 0.460  $\mu\text{m}$  particles deposited. In other words, in this concentration range the blocking effect of 0.816  $\mu\text{m}$  particles that have already been deposited onto a filter grain surface is larger than that of 0.460  $\mu\text{m}$  particles. When the surface area concentration was increased above 0.37  $\text{cm}^2/\text{mL}$ , 0.460  $\mu\text{m}$  particles were removed better compared to 0.816  $\mu\text{m}$  particles. This may be because 0.460  $\mu\text{m}$  particles utilize more surface area of filter grains for their deposition.

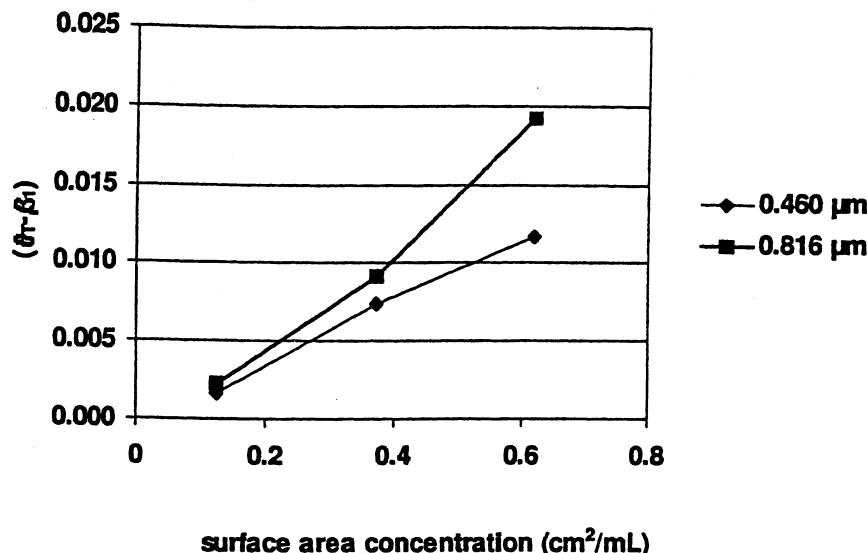
For the same reason, for monodispersed suspensions of 0.460 and 0.816  $\mu\text{m}$  particles having equal number concentrations in the range between  $56.06 \times 10^6/\text{mL}$  and  $93.43 \times 10^6/\text{mL}$ , the blocking effect of deposited particles will be more for 0.816  $\mu\text{m}$  particles. Therefore, when both surface area and number

TABLE 6  
Effect of Influent Concentration of  
Monodispersed Suspension of 0.816  $\mu\text{m}$   
Particles on  $\theta_T$  (at  $T = 120$  minutes)

Concentration (mg/L)	$\theta_T$ at $T = 120$ minutes
1.77	0.006
5.32	0.019
5.58	0.020
8.87	0.029
16.74	0.050
27.91	0.061



a)



b)

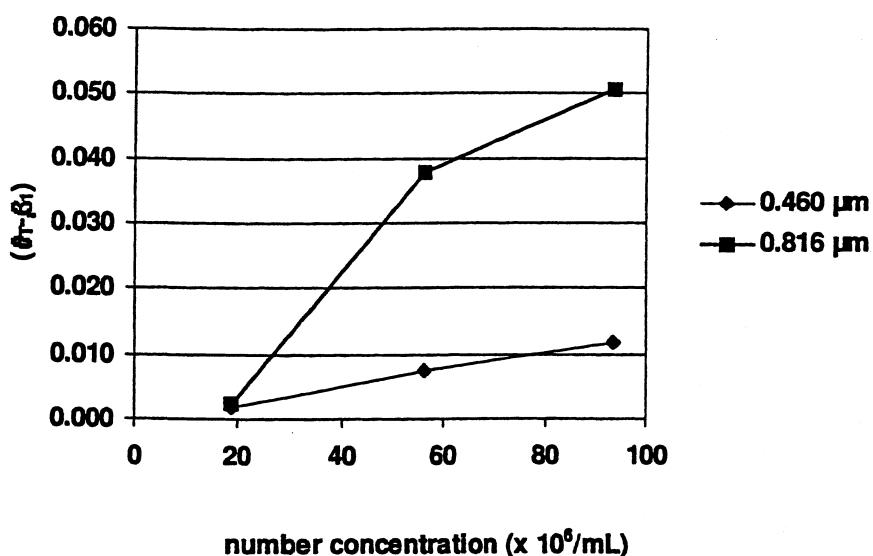


FIG. 6 Variation of  $(\theta_T - \beta_1)$  for 0.460 and 0.816  $\mu\text{m}$  particles at a) equal surface area concentration and b) equal number concentration.



concentrations were increased above a certain level, 0.460  $\mu\text{m}$  particles were removed better compared to 0.816  $\mu\text{m}$  particles because they utilized more surface area of the filter grains for deposition.

### Ionic Strength

#### *Effect of Ionic Strength on $\beta_1$*

Figures 2(d) and 2(e) show the removal efficiency of monodispersed suspensions of 0.460 and 0.816  $\mu\text{m}$  particles at different ionic strengths. Removal efficiency increased with an increase in ionic strength. Negatively charged latex particles were used in this work to study the effect of ionic strength on the surface coverage of the filter medium. When the ionic strength is varied for monodispersed suspensions of 0.460 and 0.816  $\mu\text{m}$  particles, the fraction of filter grain surface ( $\beta_1$ ) on which actual particle deposition occurs was found to increase with the ionic strength (Table 7).

#### *Effect of Ionic Strength on $\theta_T$*

The specific surface coverage,  $\theta_T$ , for the monolayer deposition of particles is shown in Fig. 7 for the filtration of monodispersed suspensions of 0.460 and 0.816  $\mu\text{m}$  particles. For both particles,  $\theta_T$  increases with an increase in the ionic strength. From the figure it can be seen that although the rate of increase of  $\theta_T$  is larger for 0.460  $\mu\text{m}$  particles, the variation of  $C/C_0$  with  $\theta_T$  is similar for both particles at ionic strengths below  $10^{-2.5}$  M KCl. However, at an ionic strength of  $10^{-2}$  M KCl, the rate of increase of  $C/C_0$  with respect to  $\theta_T$  is faster for 0.816  $\mu\text{m}$  particles.

TABLE 7  
Effect of Ionic Strength of Monodispersed Suspensions of 0.460 and 0.816  $\mu\text{m}$  Particles on  $\beta_1$  and  $\theta_T$  (at  $T = 120$  minutes)

Ionic strength, $\log[\text{KCl}]$	0.460 $\mu\text{m}$		0.816 $\mu\text{m}$	
	$\beta_1$	$\theta_T$ at $T = 120$ minutes	$\beta_1$	$\theta_T$ at $T = 120$ minutes
-6.0	0.001	0.002	0.0004	0.001
-4.0	0.001	0.003 <sup>a</sup>	0.003	0.007
-3.0	0.003	0.013	0.003	0.009
-2.5	0.005	0.020	0.005	0.013
-2.0	0.020	0.032	0.008	0.016

<sup>a</sup>  $T = 105$  minutes.

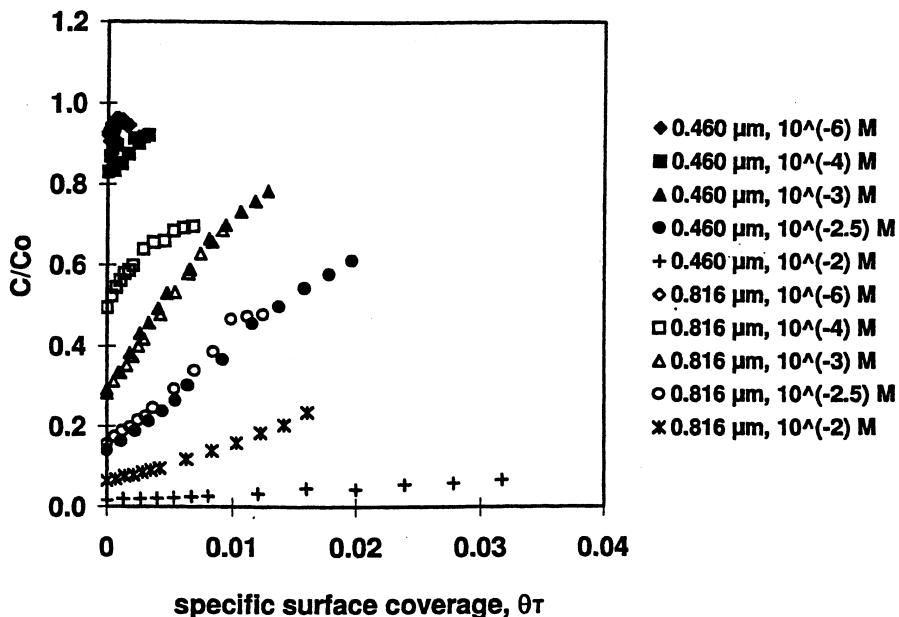


FIG. 7 Relationship between specific surface coverage and  $C/C_0$  for monodispersed suspensions of 0.460 and 0.816  $\mu\text{m}$  particles at different ionic strengths ( $L = 10 \text{ cm}$ ,  $U = 2.5 \text{ m/h}$ ).

### Comparison between $\beta_1$ and $\theta_T$

In general, the value  $(\theta_T - \beta_1)$  is larger for 0.460  $\mu\text{m}$  particles compared to 0.816  $\mu\text{m}$  particles (Fig. 8). Thus, the fraction of surface area of a filter grain utilized by 0.460  $\mu\text{m}$  particles is less compared to 0.816  $\mu\text{m}$  particles.

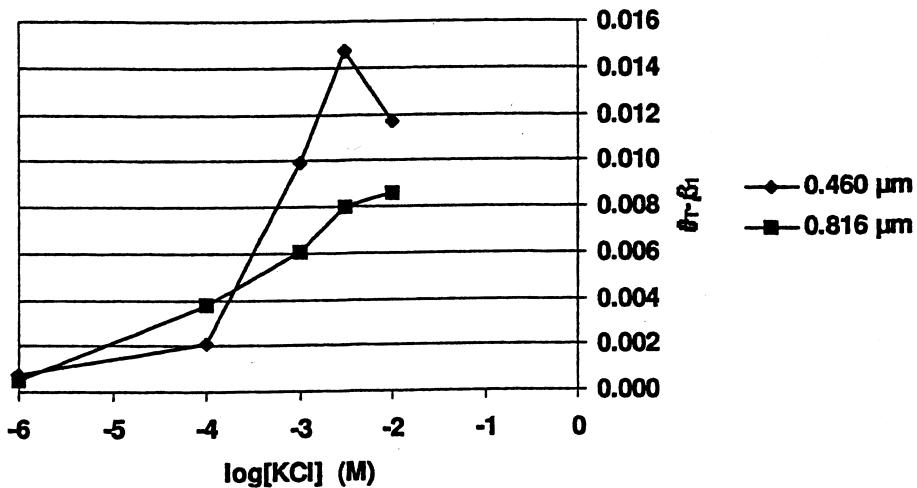


FIG. 8 Variation of  $(\theta_T - \beta_1)$  for 0.460 and 0.816  $\mu\text{m}$  particles at different ionic strengths.



This may be the reason for the faster deterioration of filter efficiency (increase in  $C/C_0$ ) for  $0.46 \mu\text{m}$  particles. However,  $(\theta_T - \beta_1)$  is nearly the same for both  $0.460$  and  $0.816 \mu\text{m}$  particles at an ionic strength of  $10^{-2} \text{ M KCl}$ , and the removal efficiency of  $0.460 \mu\text{m}$  particles is found to be better than that of  $0.816 \mu\text{m}$  particles.

## CONCLUSION

Removal of particles in a monodispersed suspension can be predicted using the model based on the analogy between adsorption and bed filtration. Several interesting observations were made on the model coefficient  $\beta_1$  that represents the fraction of a filter grain on which actual particle deposition occurs. For monodispersed suspensions of  $0.460$  and  $0.816 \mu\text{m}$  particles, under the experimental conditions used in this study,  $\beta_1$  can be related to the filtration variables as follows:

- When the mass concentration of the monodispersed suspensions was increased up to  $5 \text{ mg/L}$ , the improvement in the removal efficiency (decrease in the  $C/C_0$  values) is modeled by the increase in the fraction of surface area of a filter grain ( $\beta_1$ ) on which actual particle deposition occurs. For a given mass concentration of monodispersed suspensions (within  $5 \text{ mg/L}$ ),  $\beta_1$  is larger for suspensions having a smaller size of particle ( $0.460 \mu\text{m}$ ) and correspondingly having better removal efficiency than that of a larger particle ( $0.816 \mu\text{m}$ ). This can be attributed to the lesser blocking effect of a smaller particle compared to a larger particle in the concentration range studied.
- For equal area and number concentrations of the monodispersed suspensions, removal of  $0.816 \mu\text{m}$  particles is larger at lower concentrations (below  $56.06 \times 10^6 \text{ /mL}$  and  $0.37 \text{ cm}^2/\text{mL}$ , respectively) and smaller at higher concentrations. Thus, at lower concentrations  $\beta_1$  is larger for  $0.816 \mu\text{m}$  particles and at higher concentrations  $\beta_1$  is larger for  $0.460 \mu\text{m}$  particles, implying that the blocking effect of  $0.816 \mu\text{m}$  particles becomes prominent compared to that of  $0.460 \mu\text{m}$  particles with an increase in both number and surface area concentrations.
- When the ionic strength was increased (from  $10^{-6}$  to  $10^{-2.5} \text{ M KCl}$ ),  $\beta_1$  increased for both monodispersed suspensions. However, the blocking effect was larger for  $0.460 \mu\text{m}$  particles in this range of ionic strength. Thus, at a given ionic strength, the removal efficiency of  $0.460 \mu\text{m}$  particles during the working stages of filtration deteriorated faster than that of  $0.816 \mu\text{m}$  particles.

These observations are useful in evaluating filter performance in terms of the utilization of available surface area of the filter medium. This work



demonstrates the ability of the model to compute surface coverage at different concentrations of influent and ionic strengths of the suspension. The model can be used successfully to study the effect of other parameters such as flow rate and density of the deposit which are already explicitly incorporated in the model (9, 16–18).

Further, the level of dendrite formation of particles on filter grains during filtration can be related to the difference between  $\beta_1$  and the specific surface coverage  $\theta_T$  (the fraction of filter grain surface that is covered by particles, for a monolayer deposition, at time  $T$ ). This semiquantitative study will lead to the improvement of modeling transient stage removal of filters.

## APPENDIX

### Derivation of Equation (4)

The two terms contact efficiency ( $\eta$ ) and attachment coefficient ( $\alpha$ ) defined below are necessary in deriving Eq. (4):

$$\text{Contact efficiency of a filter grain, } \eta = \frac{\text{rate at which particles strike the filter grain}}{\text{rate at which particles flow toward the filter grain}} \quad (19)$$

$$\text{Attachment coefficient, } \alpha = \frac{\text{number of collisions which succeeded in producing adhesion}}{\text{number of collision totally occurred}} \quad (20)$$

Combining Eqs. (19) and (20):

$$\text{Number of collisions which succeeded in producing adhesion at unit time} = \alpha\eta \text{ (rate at which particles flow toward the filter grain)} \quad (21)$$

$$= \alpha\eta[(\pi/4) d_c^2 Un]$$

where  $d_c$  is the diameter of a filter grain,  $U$  is the filtration velocity, and  $n$  is the particle concentration. If one assumes that the rate of particles striking and attaching directly to the surface of the filter grain is proportional to the available surface area, the proportionality factor that can be used to find the number of particles attached on the filter grain at a particular time  $t$  is  $(N_{\max} - N_1)/N_{\max}$ , where  $N_{\max}$  is the maximum number of particles that can be retained on a filter grain and  $N_1$  is the number of particles directly attached to a filter grain at time  $t$ .

Thus, the number of particles attached to a filter grain at a particular time

$$t \text{ (Eq. 4)} = \alpha\eta[(N_{\max} - N_1)/N_{\max}][(\pi/4)d_c^2 Un]$$



## SYMBOLS

$A_0$	surface area concentration of particles ( $L^{-1}$ )
$a_c, a_p$	radius of filter grains and radius of particles, respectively ( $L$ )
$C, C_0$	particle concentration in the influent and the effluent, respectively ( $M \cdot L^{-3}$ )
$d_c, d_p$	diameter of filter grains and diameter of particles, respectively ( $L$ )
$L$	depth of the filter ( $L$ )
$N$	number of particle collectors in a unit volume of filter
$N_{\max}$	maximum number of particles retained on the surface of a filter grain
$N_0$	number concentration of particles in the influent ( $L^{-3}$ )
$N_{\text{out}}$	number concentration of particles in the effluent ( $L^{-3}$ )
$N_1$	number of particles directly attached on the filter grain (at a given time $t$ )
$N_2$	total number of particles attached to a filter grain (at a given time $t$ )
$n$	particle concentration in the suspension at a given time and depth ( $L^{-3}$ )
$n_i$	concentration of particles at the $i$ th time interval in an incremental depth, $\Delta L$ ( $L^{-3}$ )
$n_0$	influent concentration of particles at the $i$ th time interval in an incremental depth, $\Delta L$ ( $L^{-3}$ )
$t$	time (T)
$U$	approach velocity of the suspension ( $L \cdot T^{-1}$ )
$V_d, V_f$	volume of particle deposit and volume of the filter bed, respectively ( $L^3$ )
$W_d$	dry weight of the particle deposit (M)

## Greek Letters

$\alpha, \alpha_p$	attachment coefficient between particles and filter grain, and attachment coefficient between particles and particles, respectively
$\beta$	fraction of retained particles acting as particle collectors
$\beta_1$	fraction of filter grain surface available for particle deposition
$\gamma$	ratio between the number of particles attached directly to a filter grain and the total number of particles attached to a filter grain
$\Delta$	increment in filter depth or time
$\varepsilon$	porosity of a filter
$\varepsilon_d$	porosity of deposit
$\varepsilon_0$	porosity of clean bed
$\varepsilon^*$	limiting porosity



$\eta, \eta_p$	contact efficiency of a filter grain, and contact efficiency of a particle, respectively
$\eta_r$	removal efficiency of a single collector (filter grain and associated particle collectors)
$\eta_i$	removal efficiency at $i$ th time interval
$\theta_T$	specific surface coverage for monolayer deposition at time $T$ from the beginning of a filter run
$\rho$	density of the particles ( $M \cdot L^{-3}$ )
$\rho_d$	density of the deposit ( $M \cdot L^{-3}$ )
$\sigma, \sigma_{ult}$	specific deposit and ultimate specific deposit, respectively

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