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Surface Coverage of Filter Medium in Deep Bed Filtration: Mathematical Modeling and Experiments

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ABSTRACT

This paper highlights the importance of surface coverage in modeling the removal of particles in deep bed filtration. A model that considers the saturation of sites on which particle deposition occurs is used. Experimental results obtained with monodispersed suspensions of 0.46 and 0.816 μm latex particles at different influent concentrations and ionic strengths were used to calculate the fraction of filter grain surface (β_1) on which actual particle deposition occurs. This will be useful in evaluating the filter performance in terms of the utilization of available surface area of the filter medium. Further, the level of dendrite formation of particles on filter grains during filtration is expressed in terms of β_1 and the specific surface coverage, θ_T (the fraction of a filter grain surface that is covered by particles at time T , assuming that the filter grain is covered by a monolayer of particles). This can be used to compare the contribution of deposited particles in the removal efficiency of deep bed filtration for suspensions with different physical and chemical characteristics.

Key Words. Deep bed filtration; Dendrites; Filter grain surface; Ionic strength; Latex particles; Mass concentration; Number concentration; Monolayer deposition; Specific surface coverage; Surface area concentration

INTRODUCTION

Particle removal in deep bed filtration is physicochemical in nature and depends on the physical and chemical characteristics of particles, filter grain, water, and chemicals used. A number of mathematical models have been de-

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veloped to calculate particle capture in the filter (1–14). One approach has been to establish the mathematical model based on the analogy between adsorption and filtration (4, 9). In this approach the effective surface coverage of filter grains by the particles is assessed using the mathematical model for various filtration conditions. The effect of influent concentration and ionic strength of the solution on the extent of surface coverage was studied.

MATHEMATICAL FORMULATIONS

In this study the model developed based on the analogy between adsorption and filtration was used. A brief explanation of the model and the calculation method of specific surface coverage are given below. Details can be found elsewhere (9).

Mathematical Model

This model considered that there is a saturation of sites on which particle deposition occurs. Deposition sites on both the filter grain and on the retained particles are taken into account. When the deposition exceeds a limit, there is no more retention of particles within the layer under consideration. Two limits were imposed: one on the maximum number of particles that can be attached directly onto the filter grain and the other on the maximum number of particles that can be retained both on the filter grain and on the particle collectors associated with that filter grain.

The maximum number of particles deposited directly onto the surface of the filter grain is given by

$$\frac{\text{surface area of filter grain}}{\text{cross-sectional area of particle to be removed}}$$

i.e.,

$$4[d_c/d_p]^2$$

where d_c and d_p are the diameters of filter grain and particle, respectively. The above expression is based on the fact that the whole surface of a filter grain is active in the deposition of particles. But in actual fact, only a portion (β_1) of the surface of the grain will be available for the direct deposition of the particles because of the shadow effect. The maximum number of particles (N_{\max}) retained on the surface of a filter grain can therefore be given by

$$N_{\max} = 4\beta_1[d_c/d_p]^2 \quad (1)$$

The maximum number of particles associated with a filter grain (i.e., the number of particles deposited on the filter grain and on the particle collectors associated with that filter grain) is defined by defining a “limiting poros-



ity" (ε^*) term. The limiting porosity can be related to ultimate specific deposit (σ_{ult}) by

$$\varepsilon^* = \varepsilon_0 - \sigma_{ult}/(1 - \varepsilon_d) \quad (2)$$

where ε_0 is the porosity of a clean filter bed and ε_d is the porosity of deposit. The porosity of the filter ε is reduced with the progressive deposition of particles (Eq. 3), and there will not be any further deposition in the particular layer considered when its limiting value is reached.

The porosity at a given time (ε) can be related to the specific deposit (σ) at that particular time as follows:

$$\varepsilon = \varepsilon_0 - \sigma/(1 - \varepsilon_d) \quad (3)$$

Particle deposition in deep bed filtration is governed both by particle transport toward filter grains and attachment thereafter. First, particles in suspension are transported near filter grains by mechanisms such as sedimentation, interception, diffusion, inertia, and hydrodynamic effect. This is followed by the attachment of particles onto filter grains or particles already deposited onto filter grains. The attachment depends on the surface forces acting between particles and filter grains when their separation distance becomes of the order of a nanometer. The surface forces will be altered continuously with the progressive deposition of particles onto filter grains. Thus, the rate of striking of particles onto filter grains (or attachment) depends on the available surface area or deposition sites. If one assumes that the rate of particles striking and attaching directly to the surface of the filter grain is proportional to the available surface area of the filter grain, then the amount of particles attached on the filter grain at a particular time t is given by (see Appendix for the derivation of Eq. 4)

$$\alpha\eta[(N_{max} - N_1)/N_{max}](\pi/4)d_c^2Un \quad (4)$$

where α is the attachment coefficient between particles and the filter grain, η is the contact efficiency of a filter grain, N_1 is the number of particles directly attached on the filter grain at a given time t , U is the filtration velocity, and n is the particle concentration at a given time and depth. Here, it is assumed that at any given time, a fixed fraction (γ) of the total number of particles deposited (N_2) is attached directly to the filter grain, i.e.,

$$N_1 = \gamma N_2 \quad (5)$$

Further, if one assumes that the particle deposition on the particle collector is proportional to the available storage space (the available space is proportional to $\varepsilon - \varepsilon^*$), then the efficiency of the removal of particles by a particle collector can be given as

$$\alpha_p\eta_p[(\varepsilon - \varepsilon^*)/(\varepsilon_0 - \varepsilon^*)](\pi/4)d_p^2Un \quad (6)$$

where α_p is the attachment coefficient between particles and particles collectors, and η_p is the contact efficiency of a particle collector. Combining the above three equations, the removal efficiency of a single collector (filter grain and associated particle collectors on it), η_r , can be given as

$$\eta_r = \alpha\eta[(N_{\max} - \gamma N_2)/N_{\max}] + \alpha_p\eta_p N[d_p/d_c]^2[(\varepsilon - \varepsilon^*)/(\varepsilon_0 - \varepsilon^*)] \quad (7)$$

where N is the number of particle collectors in a unit volume. The rate of change of particle collectors at any time can be calculated as follows by assuming that a fraction, β , of retained particles acts as particle collectors:

$$\partial N/\partial t = \beta(\pi/4) d_c^2 U n \eta_r \quad (8)$$

From the mass balance of the suspension in a small volume element of the filter bed, the following equation can be written:

$$\partial n/\partial t + U\partial n/\partial L + (\frac{3}{2})[(1 - \varepsilon_0)/d_c] U n \eta_r = 0 \quad (9)$$

where L is the filter depth. Considering η_r and n as step functions of time and assuming steady state ($\partial n/\partial t = 0$), Eq. (9) can be written

$$\partial n/\partial L = -(\frac{3}{2})[(1 - \varepsilon_0)/d_c] n \eta_r \quad (10a)$$

By integrating Eq. (10a) between the limits 0 and ΔL for L , and n_0 and n_i for n , the following expression can be obtained:

$$n_i/n_0 = \exp[(-\frac{3}{2})(1 - \varepsilon_0) \eta_{r(i-1)}(\Delta L/d_c)] \quad (10b)$$

where n_i is the particle concentration at the i th time step, n_0 is the influent particle concentration, and ΔL is the increment in filter depth. The rate of change of the number of particle collectors at the $(i - 1)$ th time step is given by

$$(N_i - N_{i-1})/\Delta t = \beta(\pi/4) d_c^2 U n_{i-1} \eta_{r(i-1)} \quad (11)$$

Therefore, the total number of particle collectors up to the i th time can be given by

$$N_1 = \beta(\pi/4) d_c^2 U \sum_{i=1}^t n_{i-1} \eta_{r(i-1)} \Delta t \quad (12)$$

Substituting Eqs. (10) and (12) in Eq. (7), the removal efficiency at the $i - 1$ th time interval can be obtained as follows:

$$\eta_{r(i-1)} = \alpha\eta[(1 - \gamma N_2/N_{\max})] + \alpha_p\eta_p\beta \times (\pi/4) [d_p]^2 U [(\varepsilon - \varepsilon^*)/(\varepsilon_0 - \varepsilon^*)] \sum_{i=1}^t n_0 \eta_{r(i-1)} \times \Delta t \exp[(-\frac{3}{2})(1 - \varepsilon_0) \eta_{r(i-1)}(\Delta L/d_c)] \quad (13)$$



The value of the ultimate specific deposit should be known in order to calculate the maximum pore blockage or the porosity of the clogged bed (ε^*). The procedure to evaluate ε^* is given in the following section.

Calculation Procedure to Evaluate the Porosity ε^* of a Clogged Bed

The porosity deposit (ε_d), ultimate specific deposit (σ_{ult}), and ultimate porosity of the filter bed (ε^*) were experimentally measured by Vigneswaran and Chang (10). Since the ratio of effluent to influent concentration (C/C_0) will take a very long time to reach the value of unity, the filter run is terminated when the postbreakthrough stage starts. The postbreakthrough stage is defined as the period where the effluent concentration remains almost constant after a period of deterioration of effluent quality (i.e., after the breakthrough period). The deposit is washed out (when the postripening period is reached) and allowed to settle down up to a permanent volume (V_d). The dry weight of the deposit (W_d) is then measured. From these values, ε_d , σ_{ult} , and ε^* can be calculated using the following equations:

$$\varepsilon_d = 1 - (W_d/\rho_d)/V_d \quad (14)$$

$$\sigma_{ult} = (W_d/\rho_d)/V_f \quad (15)$$

where ρ_d is the density of the deposit and V_f is the volume of the filter bed. Here, the ultimate specific deposit, σ_u , is defined as the deposit up to the period of postbreakthrough per unit volume of filter. Then ε^* can be given as follows (Eq. 2):

$$\varepsilon^* = \varepsilon_0 - (\sigma_{ult})/(1 - \varepsilon_d)$$

ε_d was taken as 0.8 in this study, which is consistent with the values used in previous work with the same suspensions (9).

Specific Surface Coverage

Specific surface coverage (θ) in a time interval (Δt) is defined as the ratio between the surface area of filter grains that is covered by deposited particles in a unit bed volume in a time interval of Δt and the total surface area of filter grains in a unit bed volume. Thus, for a monolayer coverage of particles onto filter grains, θ at time T from the beginning of filtration (θ_T) can be given as (13, 15)

$$\theta_T = \{(\pi a_p^2)UN_0 a_c/[3L(1 - \varepsilon)]\} \int_0^T (1 - N_{out}/N_0)dt \quad (16)$$

where a_c and a_p are the radii of filter grains and particles, respectively, and N_0 and N_{out} are the number concentrations of particles in the influent and in the effluent, respectively. In the model discussed in the previous section, θ de-

a) dendrite formation

b) monolayer coverage

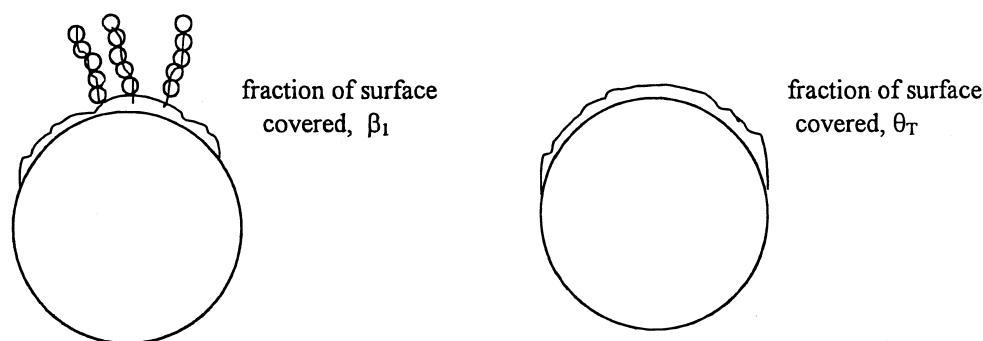


FIG. 1 Comparison of dendrite formation and monolayer coverage.

notes the fraction of filter grain surface that is directly covered by the particles. These particles and the particles deposited in dendrite forms amount to the total deposition of particles on a filter grain (Fig. 1a). On the other hand, when monolayer coverage of particles onto filter grains is assumed, the specific surface coverage θ_T denotes the fraction of filter grain surface that is directly covered by the particles, and these particles amount to the total deposition (Fig. 1b). Thus, if monolayer coverage had occurred, the value of θ_T should exceed the value of β_1 after a certain time of filtration. In the case of dendrite formation, the difference between θ_T and β_1 should represent the amount of dendrites formed.

However, monolayer coverage of filter grains is improbable for most filtration conditions, as the particles will arrive in a random manner and the coverage will build up on a declining basis with some multiple coverage occurring at the same time, possibly with the chain's formation (dendrite). In this case, θ_T will be larger than β_1 when T exceeds a particular time. From that time onward, the difference between θ_T and β_1 can be used as a measure of the amount of dendrite formed.

EXPERIMENTAL

Latex particles of 0.460 and 0.816 μm were used to prepare monodispersed suspensions of predetermined concentrations. Spherical glass beads of 0.175 mm were used as the filter medium and were packed into the cylindrical filter column to a specified depth. A downward filtration velocity of 2.5 m/h was used. Effluent turbidity (C) was measured at predetermined time intervals using a HACH turbidimeter. The zeta potential value of the influent was mea-

TABLE 1
Experimental Conditions

Effect of	Particle size (μm)	Influent concentration (mg/L)	Ionic strength, log[KCl] (M)	Filter depth (cm)
Concentration	0.460	1.0, 2.0, 3.0, 4.0, 5.0, 7.5, 10.0	-2.0	10
	0.816	1.77, 5.32, 5.58, 8.87, 16.74, 27.91	-2.0	10
Ionic strength	0.460	5.0	-6.0, -4.0, -3.0, -2.5, -2.0	10
	0.816	5.0	-6.0, -4.0, -3.0, -2.5, -2.0	10
Filter depth	0.460	1.0, 2.0, 3.0, 4.0, 5.0, 10.0	-2.0	5
	0.460	1.0, 2.0, 3.0, 4.0, 5.0, 7.5, 10.0	-2.0	10

sured using a DELSA 440. There was no particle aggregation observed in the range of influent concentration and the ionic strength used. The experimental conditions are summarized in Table 1.

RESULTS AND DISCUSSION

Experimental results obtained and the corresponding model predictions are shown in Figs. 2(a) through 2(e). Corresponding values of the model coefficient β_1 are discussed in subsequent sections. Other model parameters were kept constant at the following values: $\alpha_p\beta = 0.0001$, $\varepsilon^* = 0.30$, and $\gamma = 0.22$.

Influent Concentration

Effect of Influent Concentration on β_1

From Figs. 2(a), (b), and (c) it can be seen that the removal efficiency of 0.460 and 0.816 μm particles increases during the transient stages of filtration when the influent mass concentration of monodispersed suspensions of 0.460 and 0.816 μm particles was increased up to around 5 mg/L. When the concentration of both monodispersed suspensions of 0.460 and 0.816 μm particles was increased in this range, the effluent concentration profiles were fitted with increasing values of β_1 . The higher the concentration (up to 5 mg/L), the higher the coverage fraction of a filter grain surface (Table 2). When the concentrations of those monodispersed suspensions were increased above 5 mg/L, the working stage removal efficiency was found to decrease. Corre-

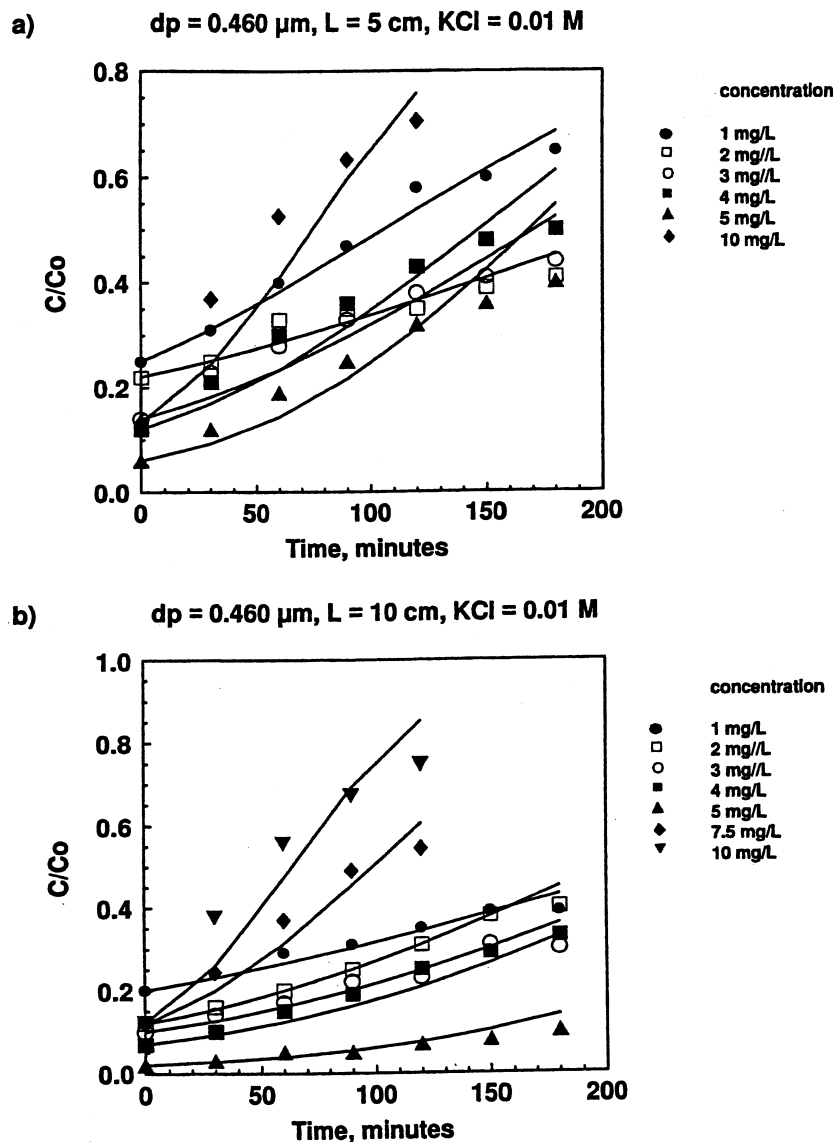


FIG. 2 Experimental data with model prediction. Symbols denote experimental data and lines denote model prediction.

spondingly, for both monodispersed suspensions the values of β_1 decreased when the influent concentration was increased above 5 mg/L.

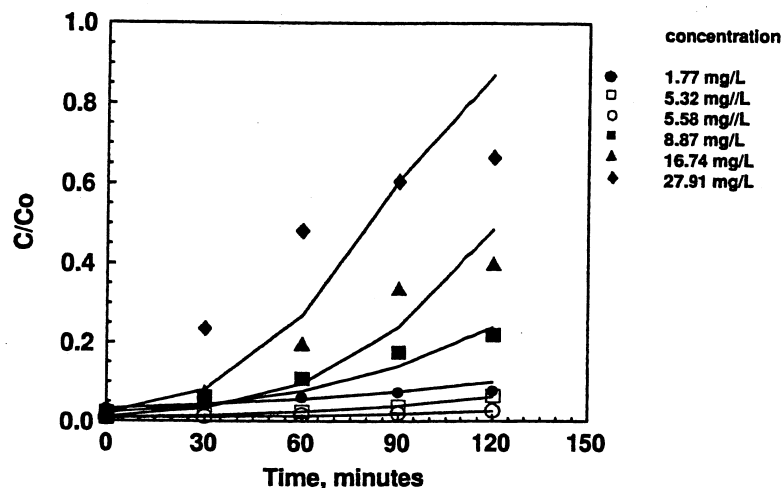
Influent concentration can also be measured as surface area concentration (A_0) and number concentration (N_0). The relationship between A_0 and N_0 with mass concentration C_0 can be written as

$$N_0 = C_0 / [(\frac{4}{3})\pi a_p^3 \rho] \quad (17)$$

$$A_0 = (4\pi a_p^2)N_0 \quad (18)$$



c) $dp = 0.816 \mu\text{m}$, $L = 10 \text{ cm}$, $\text{KCl} = 0.01 \text{ M}$



d) $dp = 0.460 \mu\text{m}$, $L = 10 \text{ cm}$, $C_0 = 5 \text{ mg/L}$

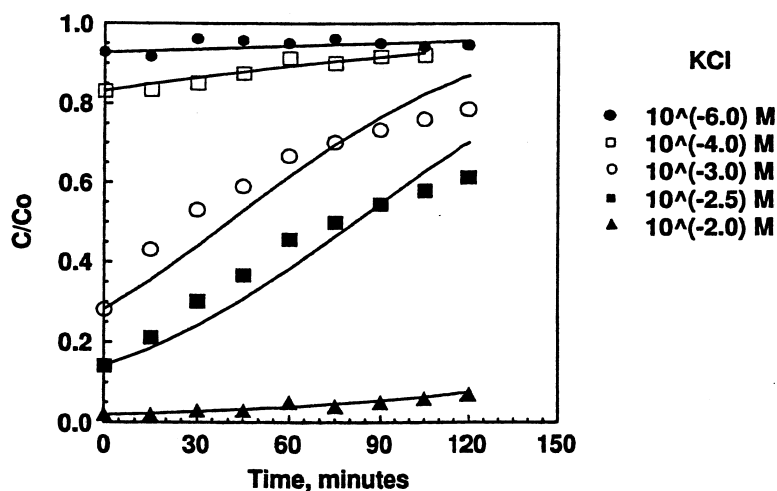


FIG. 2 Continued.

(continued)

where a_p is the radius of particles and ρ is the density of particles. Table 3 shows the relationship among mass, surface area, and number concentrations of 0.460 and 0.816 μm particles. These values were calculated using the above two equations.

For monodispersed suspensions of 0.460 and 0.816 μm particles having the same surface area concentrations of less than $0.37 \text{ cm}^2/\text{mL}$, the fraction of a filter grain surface (β_1) on which actual particle deposition occurs is larger for 0.816 μm particles (Table 4). Thus, for equal area concentrations of less than $0.37 \text{ cm}^2/\text{mL}$, the removal of 0.816 μm particles is better than that of 0.460



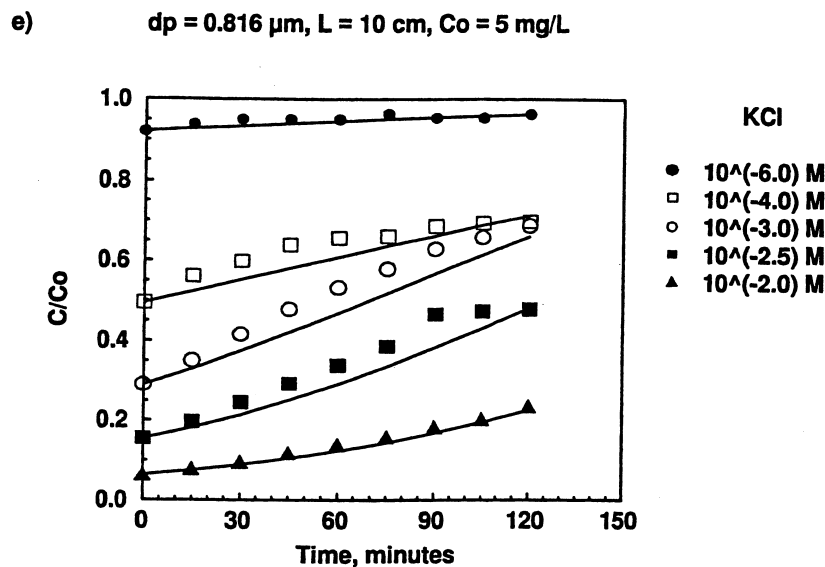


FIG. 2 Continued.

μm particles (i.e., smaller C/C_0 values for $0.816 \mu\text{m}$ particles compared to $0.460 \mu\text{m}$ particles during filtration).

For equal surface area concentrations between 0.37 and $0.62 \text{ cm}^2/\text{mL}$, β_1 is larger for $0.460 \mu\text{m}$ particles and correspondingly the removal of $0.460 \mu\text{m}$ particles is better than that of $0.816 \mu\text{m}$ particles.

Similarly, for monodispersed suspensions of 0.460 and $0.816 \mu\text{m}$ particles having the same number concentrations of less than $56.06 \times 10^6/\text{mL}$, the frac-

TABLE 2
Value of β_1 at Different Mass Concentrations of Monodispersed Suspensions of 0.460 and $0.816 \mu\text{m}$ Particles

0.460 μm		0.816 μm	
Concentration (mg/L)	β_1	Concentration (mg/L)	β_1
1.0	0.003	1.77	0.004
2.0	0.005	5.32	0.010
3.0	0.009	5.58	0.018
4.0	0.012	8.87	0.010
5.0	0.020	16.74	0.012
7.5	0.009	27.91	0.010
10.0	0.008		



TABLE 3
Variation of Surface Area and Number Concentration with the Influent Concentration

Influent concentration (mg/L)	0.460 μm Particles		0.816 μm Particles	
	cm^2/mL	No. $\times 10^6/\text{mL}$	cm^2/mL	No. $\times 10^6/\text{mL}$
1.00	0.124	18.69	0.070	3.35
1.77	0.220	33.08	0.124	5.93
3.00	0.373	56.06	0.210	10.04
5.00	0.621	93.44	0.350	16.74
5.32	0.661	99.42	0.373	17.81
5.58	0.693	104.27	0.391	18.68
8.87	1.102	165.75	0.621	29.69
16.74	2.080	312.82	1.172	56.04
27.90	3.466	521.37	1.954	93.40

* Equal surface area concentrations and equal number concentrations of 0.46 and 0.816 μm particles are shown in bold.

tion of a filter grain surface (β_1) on which actual particle deposition occurs is larger for 0.816 μm particles. For equal number concentrations between $56.06 \times 10^6/\text{mL}$ and $93.43 \times 10^6/\text{mL}$, β_1 is larger for 0.460 μm particles. Thus, for equal number concentrations less than $56.06 \times 10^6/\text{mL}$, the removal of 0.816 μm particles is better than that of 0.460 μm particles, and for equal number concentrations between $56.06 \times 10^6/\text{mL}$ and $93.43 \times 10^6/\text{mL}$, removal of 0.460 μm particles is better than that of 0.816 μm particles.

Effect of Influent Concentration on θ_T

Figures 3 and 4 show the relationship between the specific surface coverage θ_T and C/C_0 for equal surface area concentrations and equal number concentrations, respectively. These figures imply that for both monodispersed suspensions, if the filter grains are covered by a monolayer of particles, then

TABLE 4
Values of β_1 at Different Surface Area and Number Concentrations of the Monodispersed Suspensions of 0.460 and 0.816 μm Particles

Surface area concentration (cm^2/mL)	β_1		Number concentration (no. $\times 10^6/\text{mL}$)	β_1	
	0.460 μm	0.816 μm		0.460 μm	0.816 μm
0.124	0.003	0.004	18.69	0.003	0.018
0.373	0.009	0.010	56.06	0.009	0.012
0.621	0.020	0.010	93.44	0.020	0.010

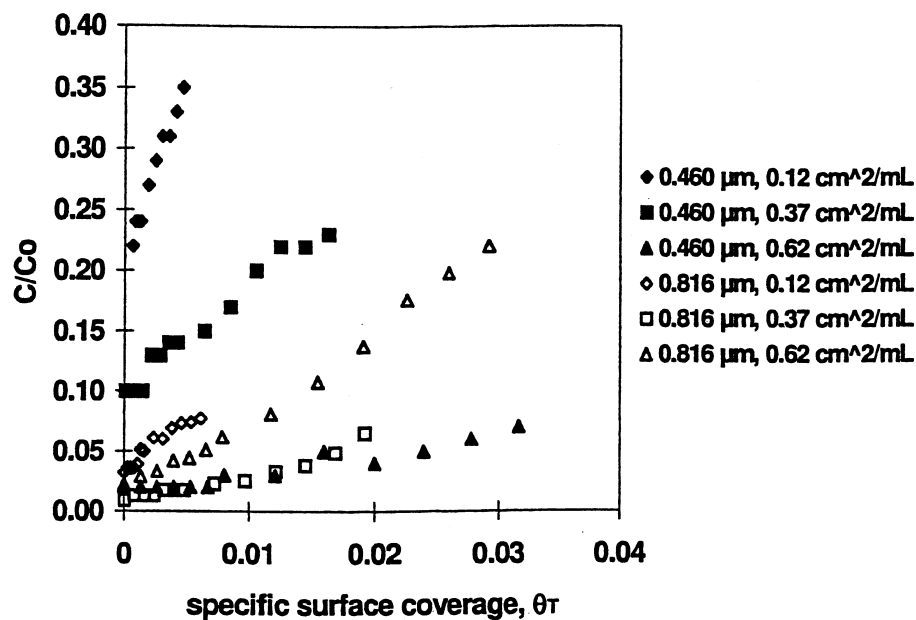


FIG. 3 Relationship between specific surface coverage and C/C_0 for monodispersed suspensions of 0.460 and 0.816 μm particles at equal surface area concentrations ($L = 10$ cm, $\text{KCl} = 0.01$ M).

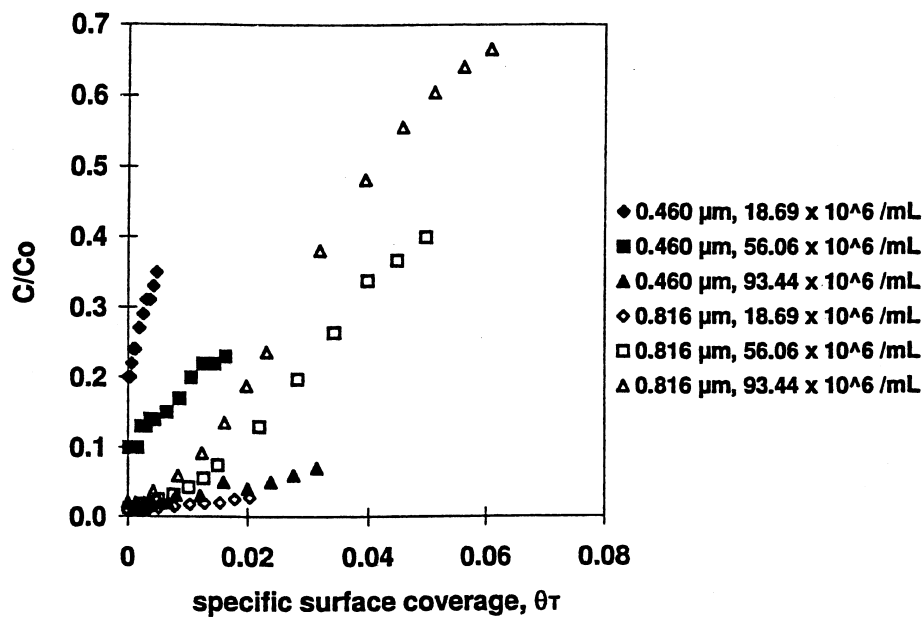


FIG. 4 Relationship between specific surface coverage and C/C_0 for monodispersed suspensions of 0.460 and 0.816 μm particles at equal number concentrations ($L = 10$ cm, $\text{KCl} = 10^{-2}$ M).



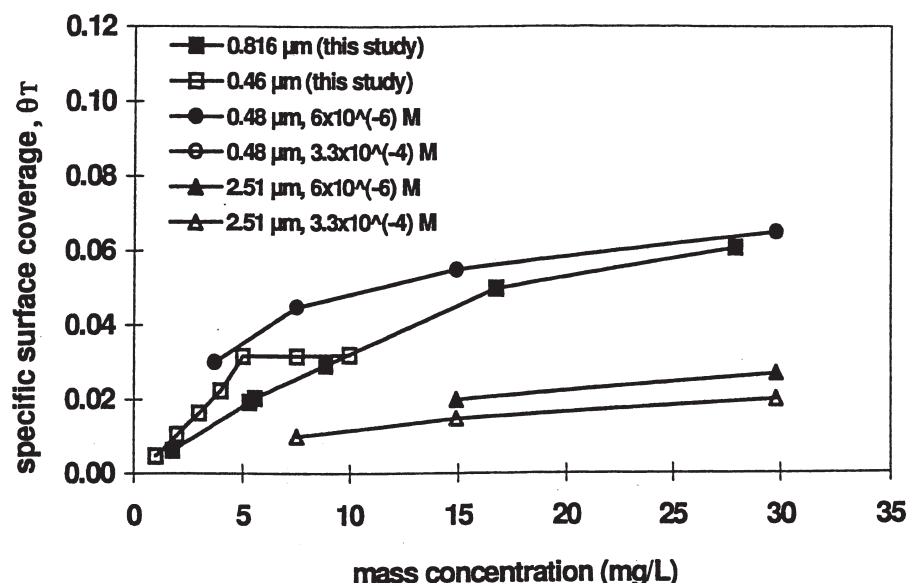


FIG. 5 Variation of specific surface coverage with influent concentration [this study: $T = 120$ minutes, $L = 10$ cm, $KCl = 0.01$ M, $U = 2.5$ m/h; previous study by Liu et al. (14): $T = 80$ minutes, $L = 10$ cm for $0.48 \mu\text{m}$ particles and 14.2 cm for $2.51 \mu\text{m}$ particles, $U = 3.6$ m/h].

at a given time T the value of θ_T is larger for a suspension having a larger surface area concentration or number concentrations. However, when the number concentrations of 0.460 and $0.816 \mu\text{m}$ particles are kept constant in the monodispersed suspensions, θ_T is larger for $0.816 \mu\text{m}$ particles compared to $0.460 \mu\text{m}$ particles. But for monodispersed suspensions with equal surface area concentrations, θ_T was similar for both 0.816 and $0.460 \mu\text{m}$ particles.

The values obtained for θ_T from the filtration experimental results (at 120 minutes of filtration) for 0.460 and $0.816 \mu\text{m}$ particles at 0.01 M KCl ionic strength are shown in Fig. 5. The results from a previous study with two different sizes of particles at two different ionic strength values are also presented for comparison (14) (at $T = 80$ minutes). Suspensions having larger particles tend to show less specific surface coverage at a particular time when compared with suspensions having smaller particles of the same mass concentration.

Comparison between β_1 and θ_T

The values of θ_T (at $T = 120$ minutes of filtration) obtained for different concentrations are presented in Tables 5 and 6. For monodispersed suspensions of 0.460 and $0.816 \mu\text{m}$ particles having an equal surface area concentration, the difference between θ_T and β_1 ($\theta_T - \beta_1$) is larger for $0.816 \mu\text{m}$ particles in the concentration range between 0.37 and $0.62 \text{ cm}^2/\text{mL}$ (Fig. 6). For this concen-



TABLE 5
Effect of Influent Concentration of Monodispersed Suspension
of 0.460 μm Particles on θ_T (at $T = 120$ minutes)

Concentration (mg/L)	θ_T at $T = 120$ minutes	
	$L = 5$ cm	$L = 10$ cm
1.0	0.008	0.005
2.0	0.018	0.010
3.0	0.028	0.016
4.0	0.038	0.022
5.0	0.053	0.032
7.5	—	0.032
10.0	0.067	0.032

tration range it is reasonable to conclude that 0.816 μm particles did not deposit directly onto a filter grain surface to the extent with which the 0.460 μm particles deposited. In other words, in this concentration range the blocking effect of 0.816 μm particles that have already been deposited onto a filter grain surface is larger than that of 0.460 μm particles. When the surface area concentration was increased above 0.37 cm^2/mL , 0.460 μm particles were removed better compared to 0.816 μm particles. This may be because 0.460 μm particles utilize more surface area of filter grains for their deposition.

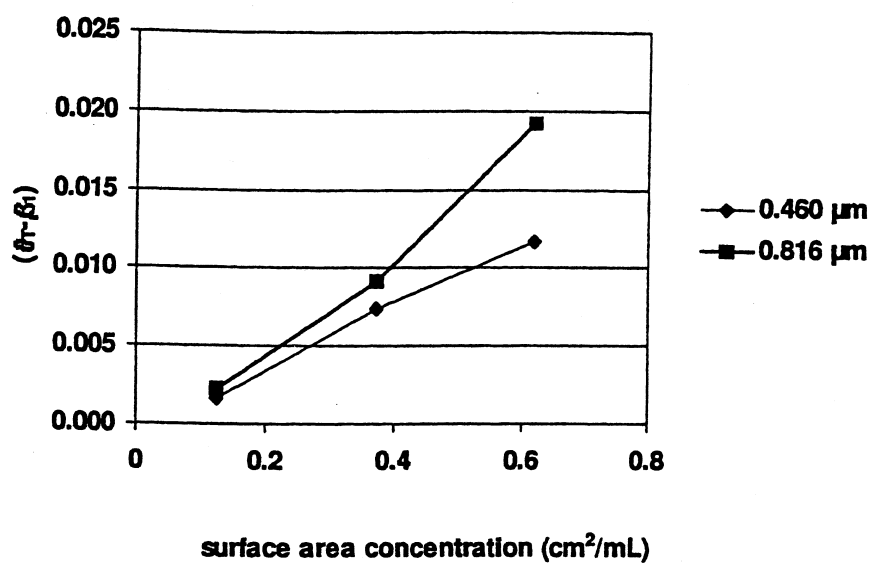
For the same reason, for monodispersed suspensions of 0.460 and 0.816 μm particles having equal number concentrations in the range between $56.06 \times 10^6/\text{mL}$ and $93.43 \times 10^6/\text{mL}$, the blocking effect of deposited particles will be more for 0.816 μm particles. Therefore, when both surface area and number

TABLE 6
Effect of Influent Concentration of
Monodispersed Suspension of 0.816 μm
Particles on θ_T (at $T = 120$ minutes)

Concentration (mg/L)	θ_T at $T = 120$ minutes
1.77	0.006
5.32	0.019
5.58	0.020
8.87	0.029
16.74	0.050
27.91	0.061



a)



b)

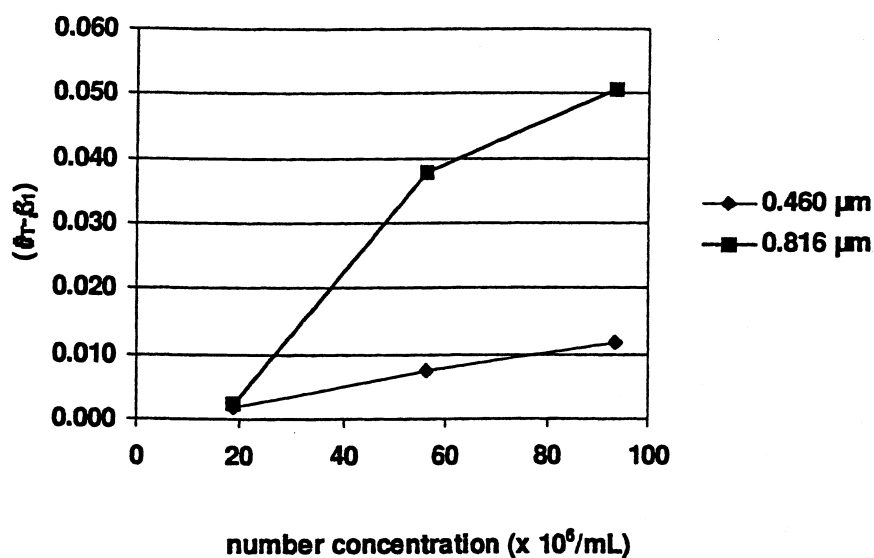


FIG. 6 Variation of $(\theta_T - \beta_1)$ for 0.460 and 0.816 μm particles at a) equal surface area concentration and b) equal number concentration.



concentrations were increased above a certain level, 0.460 μm particles were removed better compared to 0.816 μm particles because they utilized more surface area of the filter grains for deposition.

Ionic Strength

Effect of Ionic Strength on β_1

Figures 2(d) and 2(e) show the removal efficiency of monodispersed suspensions of 0.460 and 0.816 μm particles at different ionic strengths. Removal efficiency increased with an increase in ionic strength. Negatively charged latex particles were used in this work to study the effect of ionic strength on the surface coverage of the filter medium. When the ionic strength is varied for monodispersed suspensions of 0.460 and 0.816 μm particles, the fraction of filter grain surface (β_1) on which actual particle deposition occurs was found to increase with the ionic strength (Table 7).

Effect of Ionic Strength on θ_T

The specific surface coverage, θ_T , for the monolayer deposition of particles is shown in Fig. 7 for the filtration of monodispersed suspensions of 0.460 and 0.816 μm particles. For both particles, θ_T increases with an increase in the ionic strength. From the figure it can be seen that although the rate of increase of θ_T is larger for 0.460 μm particles, the variation of C/C_0 with θ_T is similar for both particles at ionic strengths below $10^{-2.5}$ M KCl. However, at an ionic strength of 10^{-2} M KCl, the rate of increase of C/C_0 with respect to θ_T is faster for 0.816 μm particles.

TABLE 7
Effect of Ionic Strength of Monodispersed Suspensions of 0.460 and 0.816 μm Particles on β_1 and θ_T (at $T = 120$ minutes)

Ionic strength, log[KCl]	0.460 μm		0.816 μm	
	β_1	θ_T at $T = 120$ minutes	β_1	θ_T at $T = 120$ minutes
-6.0	0.001	0.002	0.0004	0.001
-4.0	0.001	0.003 ^a	0.003	0.007
-3.0	0.003	0.013	0.003	0.009
-2.5	0.005	0.020	0.005	0.013
-2.0	0.020	0.032	0.008	0.016

^a $T = 105$ minutes.



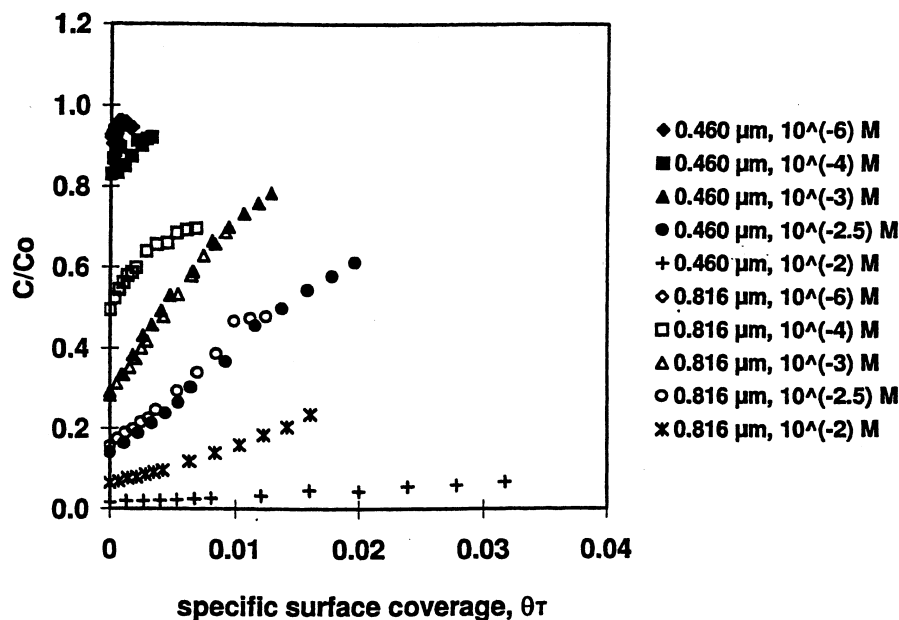


FIG. 7 Relationship between specific surface coverage and C/C_0 for monodispersed suspensions of 0.460 and 0.816 μm particles at different ionic strengths ($L = 10$ cm, $U = 2.5$ m/h).

Comparison between β_1 and θ_T

In general, the value $(\theta_T - \beta_1)$ is larger for 0.460 μm particles compared to 0.816 μm particles (Fig. 8). Thus, the fraction of surface area of a filter grain utilized by 0.460 μm particles is less compared to 0.816 μm particles.

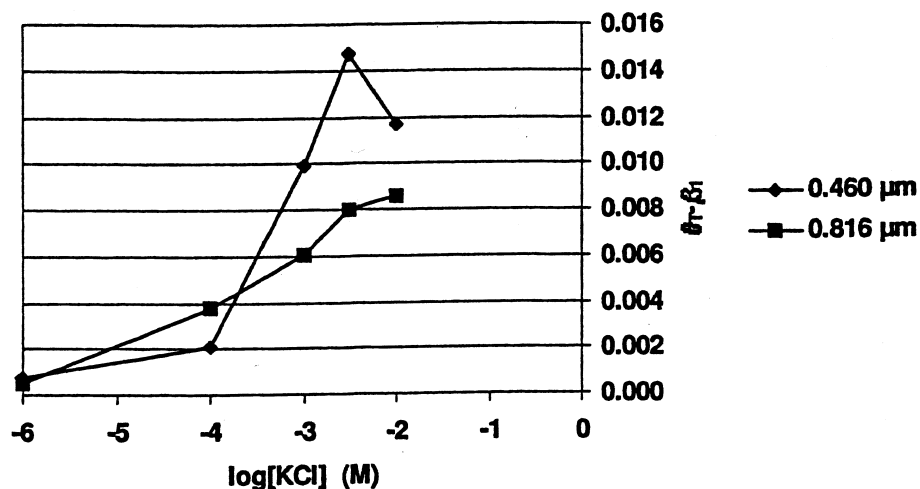


FIG. 8 Variation of $(\theta_T - \beta_1)$ for 0.460 and 0.816 μm particles at different ionic strengths.

This may be the reason for the faster deterioration of filter efficiency (increase in C/C_0) for 0.46 μm particles. However, $(\theta_T - \beta_1)$ is nearly the same for both 0.460 and 0.816 μm particles at an ionic strength of 10^{-2} M KCl, and the removal efficiency of 0.460 μm particles is found to be better than that of 0.816 μm particles.

CONCLUSION

Removal of particles in a monodispersed suspension can be predicted using the model based on the analogy between adsorption and bed filtration. Several interesting observations were made on the model coefficient β_1 that represents the fraction of a filter grain on which actual particle deposition occurs. For monodispersed suspensions of 0.460 and 0.816 μm particles, under the experimental conditions used in this study, β_1 can be related to the filtration variables as follows:

- When the mass concentration of the monodispersed suspensions was increased up to 5 mg/L, the improvement in the removal efficiency (decrease in the C/C_0 values) is modeled by the increase in the fraction of surface area of a filter grain (β_1) on which actual particle deposition occurs. For a given mass concentration of monodispersed suspensions (within 5 mg/L), β_1 is larger for suspensions having a smaller size of particle (0.460 μm) and correspondingly having better removal efficiency than that of a larger particle (0.816 μm). This can be attributed to the lesser blocking effect of a smaller particle compared to a larger particle in the concentration range studied.
- For equal area and number concentrations of the monodispersed suspensions, removal of 0.816 μm particles is larger at lower concentrations (below $56.06 \times 10^6/\text{mL}$ and $0.37 \text{ cm}^2/\text{mL}$, respectively) and smaller at higher concentrations. Thus, at lower concentrations β_1 is larger for 0.816 μm particles and at higher concentrations β_1 is larger for 0.460 μm particles, implying that the blocking effect of 0.816 μm particles becomes prominent compared to that of 0.460 μm particles with an increase in both number and surface area concentrations.
- When the ionic strength was increased (from 10^{-6} to $10^{-2.5}$ M KCl), β_1 increased for both monodispersed suspensions. However, the blocking effect was larger for 0.460 μm particles in this range of ionic strength. Thus, at a given ionic strength, the removal efficiency of 0.460 μm particles during the working stages of filtration deteriorated faster than that of 0.816 μm particles.

These observations are useful in evaluating filter performance in terms of the utilization of available surface area of the filter medium. This work



demonstrates the ability of the model to compute surface coverage at different concentrations of influent and ionic strengths of the suspension. The model can be used successfully to study the effect of other parameters such as flow rate and density of the deposit which are already explicitly incorporated in the model (9, 16–18).

Further, the level of dendrite formation of particles on filter grains during filtration can be related to the difference between β_1 and the specific surface coverage θ_T (the fraction of filter grain surface that is covered by particles, for a monolayer deposition, at time T). This semiquantitative study will lead to the improvement of modeling transient stage removal of filters.

APPENDIX

Derivation of Equation (4)

The two terms contact efficiency (η) and attachment coefficient (α) defined below are necessary in deriving Eq. (4):

$$\text{Contact efficiency of a filter grain, } \eta = \frac{\text{rate at which particles strike the filter grain}}{\text{rate at which particles flow toward the filter grain}} \quad (19)$$

$$\text{Attachment coefficient, } \alpha = \frac{\text{number of collisions which succeeded in producing adhesion}}{\text{number of collision totally occurred}} \quad (20)$$

Combining Eqs. (19) and (20):

$$\begin{aligned} \text{Number of collisions which succeeded in producing adhesion at} \\ \text{unit time} &= \alpha\eta (\text{rate at which particles flow toward the filter grain}) \quad (21) \\ &= \alpha\eta[(\pi/4)d_c^2Un] \end{aligned}$$

where d_c is the diameter of a filter grain, U is the filtration velocity, and n is the particle concentration. If one assumes that the rate of particles striking and attaching directly to the surface of the filter grain is proportional to the available surface area, the proportionality factor that can be used to find the number of particles attached on the filter grain at a particular time t is $(N_{\max} - N_1)/N_{\max}$, where N_{\max} is the maximum number of particles that can be retained on a filter grain and N_1 is the number of particles directly attached to a filter grain at time t .

Thus, the number of particles attached to a filter grain at a particular time

$$t \text{ (Eq. 4)} = \alpha\eta[(N_{\max} - N_1)/N_{\max}][(\pi/4)d_c^2Un]$$



SYMBOLS

A_0	surface area concentration of particles (L^{-1})
a_c, a_p	radius of filter grains and radius of particles, respectively (L)
C, C_0	particle concentration in the influent and the effluent, respectively ($M \cdot L^{-3}$)
d_c, d_p	diameter of filter grains and diameter of particles, respectively (L)
L	depth of the filter (L)
N	number of particle collectors in a unit volume of filter
N_{\max}	maximum number of particles retained on the surface of a filter grain
N_0	number concentration of particles in the influent (L^{-3})
N_{out}	number concentration of particles in the effluent (L^{-3})
N_1	number of particles directly attached on the filter grain (at a given time t)
N_2	total number of particles attached to a filter grain (at a given time t)
n	particle concentration in the suspension at a given time and depth (L^{-3})
n_i	concentration of particles at the i th time interval in an incremental depth, ΔL (L^{-3})
n_0	influent concentration of particles at the i th time interval in an incremental depth, ΔL (L^{-3})
t	time (T)
U	approach velocity of the suspension ($L \cdot T^{-1}$)
V_d, V_f	volume of particle deposit and volume of the filter bed, respectively (L^3)
W_d	dry weight of the particle deposit (M)

Greek Letters

α, α_p	attachment coefficient between particles and filter grain, and attachment coefficient between particles and particles, respectively
β	fraction of retained particles acting as particle collectors
β_1	fraction of filter grain surface available for particle deposition
γ	ratio between the number of particles attached directly to a filter grain and the total number of particles attached to a filter grain
Δ	increment in filter depth or time
ε	porosity of a filter
ε_d	porosity of deposit
ε_0	porosity of clean bed
ε^*	limiting porosity



η, η_p	contact efficiency of a filter grain, and contact efficiency of a particle, respectively
η_r	removal efficiency of a single collector (filter grain and associated particle collectors)
η_i	removal efficiency at i th time interval
θ_T	specific surface coverage for monolayer deposition at time T from the beginning of a filter run
ρ	density of the particles ($M \cdot L^{-3}$)
ρ_d	density of the deposit ($M \cdot L^{-3}$)
σ, σ_{ult}	specific deposit and ultimate specific deposit, respectively

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